College Algebra Students' Perceptions of Exam Errors and the Problem-Solving Process

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Research Team
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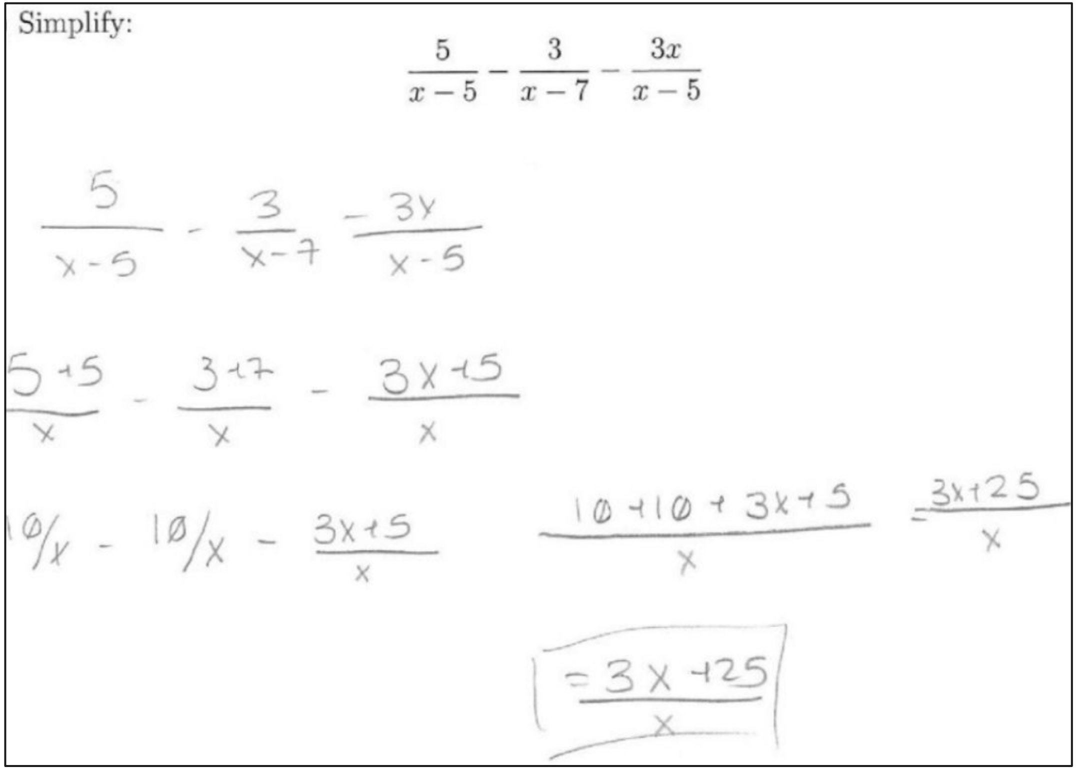
What did the student do?


Simplify:

$$\frac{5}{x-5} - \frac{3}{x-7} - \frac{3x}{x-5}$$

$$\frac{5}{x-5} - \frac{3}{x-7} - \frac{3x}{x-5}$$

$$\frac{5+5}{x} - \frac{3+7}{x} - \frac{3x+5}{x}$$

$$10/x - 10/x - \frac{3x+5}{x} \quad \frac{10+10+3x+5}{x} = \frac{3x+25}{x}$$

$$= \frac{3x+25}{x}$$

Where did they have trouble?

Did they have trouble with

1) Identifying what they were supposed to do?
2) Identifying the appropriate strategy?
3) Carrying out the appropriate strategy?
4) Checking their result?

Do you think this error is a “simple” one?

STEM students starting in College Algebra

- Although there have been a long-standing calls for more STEM majors (e.g., 2012 PCAST report)
- Some students do not arrive at university with necessary prerequisites to start in Calculus I.
- Students who begin their STEM pathway in College Algebra have little chance of success (i.e., making it through calculus sequence).
  (Herriott & Dunbar, 2009)

So I found myself coordinating College Algebra...

- Students struggled with arithmetic.
- Students struggled with introductory algebra concepts.
- A big issue we found:

  Students did not know *how*, *what*, *when*, *where* to study.

  Students struggled with knowing *what they knew* and *what they did not know*… they did not actively reflect on their work/knowledge.

Self-Reflection & Error Analysis

- Self-reflection is often a missing component of a student's academic habits.
- How often do your students 1) go back through all aspects of their coursework and exams identifying their errors and why they made them, and then 2) address the issues that led to these errors?
- Students DO learn from analyzing their errors, but they often do not go through this process and instructors do not always actively encourage it.

Errors

- Analyzing and understanding errors is part of the learning process.
  (Heinze, 2005)
- Errors are perceived negatively by both teachers and students.
  (Borasi, 1994; Kyaruzi et al., 2020)
- Using errors as learning opportunities does not often occur as a part of regular mathematics instruction.
  (Borasi, 1994; Heinze & Reiss, 2007)
- As a result, we often just show students the correct way to work a problem. Correct solutions are easier to read than those containing errors.
  (Kyaruzi et al., 2020)

Context: College Algebra

Context: College Algebra, 420 students

Co-course: College Algebra with “support” (CA-S)

- For students identified through multiple measures (e.g., high school grades, college entrance exams) as needing additional support due to weaker prerequisite backgrounds.
- Met in smaller sections of size ~30 and
  - provided additional practice on College Algebra content and
  - helped students develop effective study strategies and promote reflection.

Post Exam Reflection

After exam 1, we asked students: *Are you happy with your exam score? Why/why not?*

"I believed I would do better but I had **small** mistakes."

"I thought I did well but I made **simple** mistakes to get this grade."

"I made simple **careless** mistakes."

“It was a simple mistake.”

What do students mean by this?

We wanted to better understand why students identified mistakes as being "simple" or "not simple"

- All CA-S students were invited to participate in a voluntary interview following exam 1.
- Incentive: one-on-one exam review session with the interviewer following the interview. (*Free tutoring!*)
- n=8 CA-S students volunteered.
- Interviews: 30 minutes

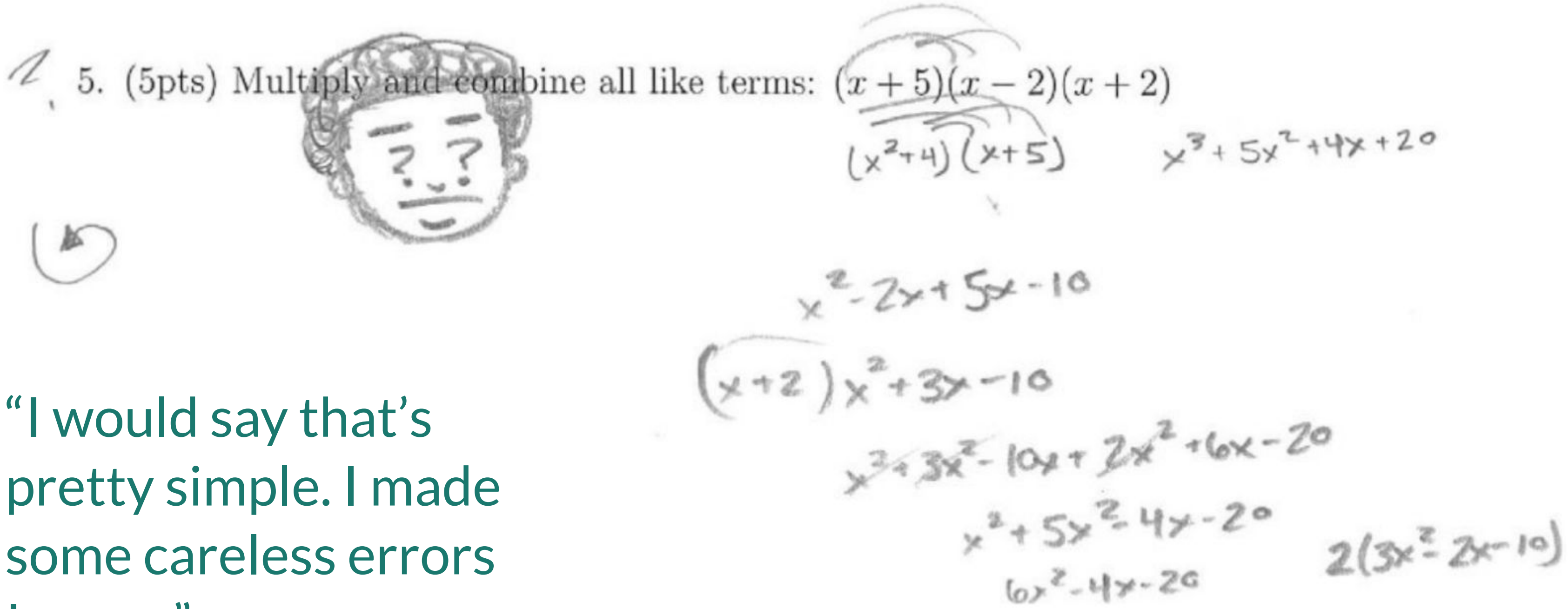
So where do these mistakes occur?

To understand students' perceptions of their mistakes within the context of problem-solving, we

1. Conducted interviews with students as they talked through their exam mistakes. During interviews students were asked *"Is this mistake simple or not simple? Why?"*
2. Adapted the Carlson and Bloom (2005) problem-solving framework as an analytical tool to understand at what stage in the problem-solving process that mistakes occurred and if there were patterns that we found with students' categorization of simple/not simple.

How would you classify this student's error?

Simple? Not simple? Why?



5. (5pts) Multiply and combine all like terms: $(x+5)(x-2)(x+2)$

$(x^2+4)(x+5)$ $x^3+5x^2+4x+20$

$x^2-2x+5x-10$

$(x+2)\,x^2+3x-10$

$x^3+3x^2-10x+2x^2+6x-20$

$x^2+5x^2-4x-20$

$6x^2-4x-20$ $2(3x^2-2x-10)$

“I would say that's pretty simple. I made some careless errors I guess.”

What do you think a "simple" mistake is?

Talk to your neighbor...

Our definition of a simple mistake:

A simple mistake was one that “could be made accidentally, would likely not be repeated, or violated a mathematical convention rather than a rule (i.e., not reducing coefficients)” (Ryals et al, 2020, p. 495).

A not simple mistake emerges from a “lack of conceptual understanding” (p. 495).

When do mistakes occur?

Carlson and Bloom (2005) provide a framework that captures metacognitive aspects of a learner's process during a distinct problem-solving task. They present a 4-stage process that

- Connects cognitive and metacognitive demands of problem-solving;
- Describes mathematicians' use of resources, application of heuristics, and behaviors related to affect and monitoring during stages of the problem-solving process.

Adapted Cyclic Problem-Solving Framework

Orienting	Planning	Executing	Checking
Successful orientation: Student understands initial state, goal state, and problem type.	Successful planning: Once a student is successfully oriented, they identify a viable approach to solving the problem.	Successful execution: Student has correct approach to solving the problem and implements the plan with no mistakes.	Successful checking: [Unable to document if answer was correct.]
Unsuccessful orientation: Student is unable to identify one or more of initial state, goal state, and problem type.	Unsuccessful planning: Student is unable to identify a viable solution to solving the problem.	Unsuccessful execution: Student is unable to completely carry out the process for solving the problem.	Unsuccessful checking: Student attributes mistakes from another phase not being caught.

Adapted from Carlson, M. P., & Bloom, I. (2005)

For example:
Multiply and combine all like terms

$$(x+5)(x-2)(x+2)$$

Initial Problem State: Product of 3 binomials

Goal State: Result is a distributed expression in which all like terms are combined.

Overall Plan: Multiply two linear expressions to obtain quadratic; then multiply quadratic and linear.

- A mistake in **orienting** typically results in a student not knowing what the initial and/or goal states are.
- A mistake in **planning** involves not having overall plan to multiply exactly 2 binomial expressions first and then multiply the third binomial by the resulting quadratic. This is often exhibited by multiplying everything all at once.
- An **executing** mistake occurs when the overall plan is carried out, but there are errors in multiplication or combining like terms.

(Orienting)

Multiply and combine all like terms

$$(x+5)(x-2)(x+2)$$

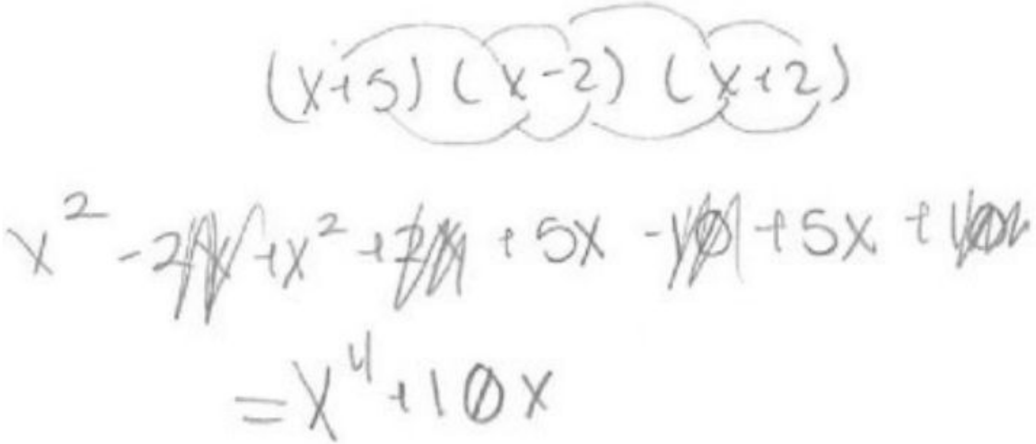
- A mistake in orienting typically results in a student not knowing what the initial and/or goal states are.
- The 8 interviewed students did not have issues orienting to this problem, but a typical orienting statement would be:

"I didn't really know how to approach the problem, I didn't know the steps and I just yeah..."

(Planning)

Multiply and combine all like terms

(x+5)(x-2)(x+2)



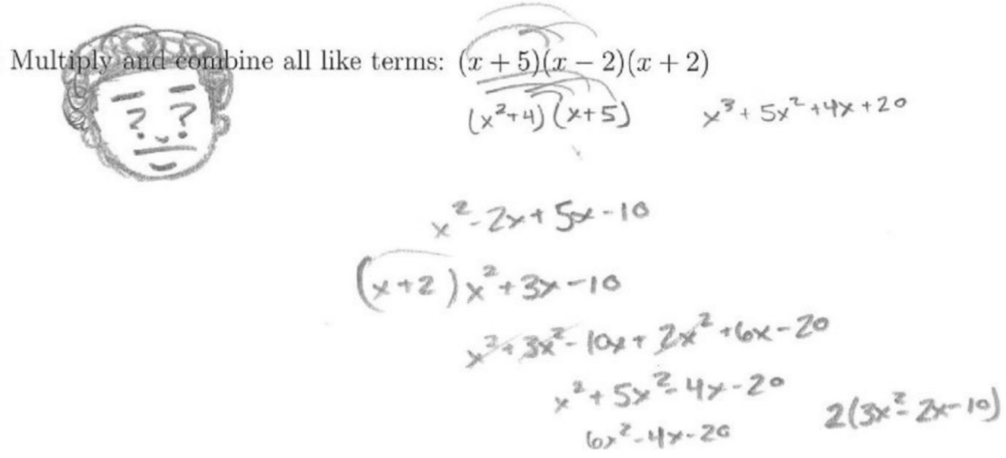
- A mistake in **planning** involves not having overall plan to multiply exactly 2 binomial expressions first and then multiply the third binomial by the resulting quadratic. This is often exhibited by multiplying everything all at once.

Student AY: "I completely spaced on that...didn't know the steps because I tried to do all three instead of just the first two and then taking that answer into the last....Probably not simple cuz like I didn't remember how to do it the right way."

(Executing)

Multiply and combine all like terms

(x+5)(x-2)(x+2)



- An executing mistake occurs when the overall plan is carried out, but there are errors in multiplication or combining like terms.

Student RG: “I made some careless errors, I guess....”

Is there a pattern or relationship between students' categorization of mistakes as *simple* or *not simple* and the problem-solving phase in which they describe the mistakes occurring?

How do students' categorization of mistakes align with that of researchers?

Note: I will be using the term "mistakes" most of the time from this point and forward, as that is the term that most students used.

Why do we care?

- This allows us to better understand how students are identifying these mistakes and how it may differ from our perspectives.
- The help and support we offer may be misaligned with their needs. For example, we may make recommendations about test taking strategies that may not be suited for addressing their needs.
- Knowing students perceptions of their mistakes helps us better understand how we might help them.

The Data:

1) Written work: A student’s completed exam 1
2) Interview: Video and audio recording of student discussing each mistake, their classification of simple/not simple, and why they make such a classification for that mistake.

Student: Oh yeah. I remember this one. Yeah I do this completely wrong. Um haha I know you can’t cancel that way. I don’t know why I did that….

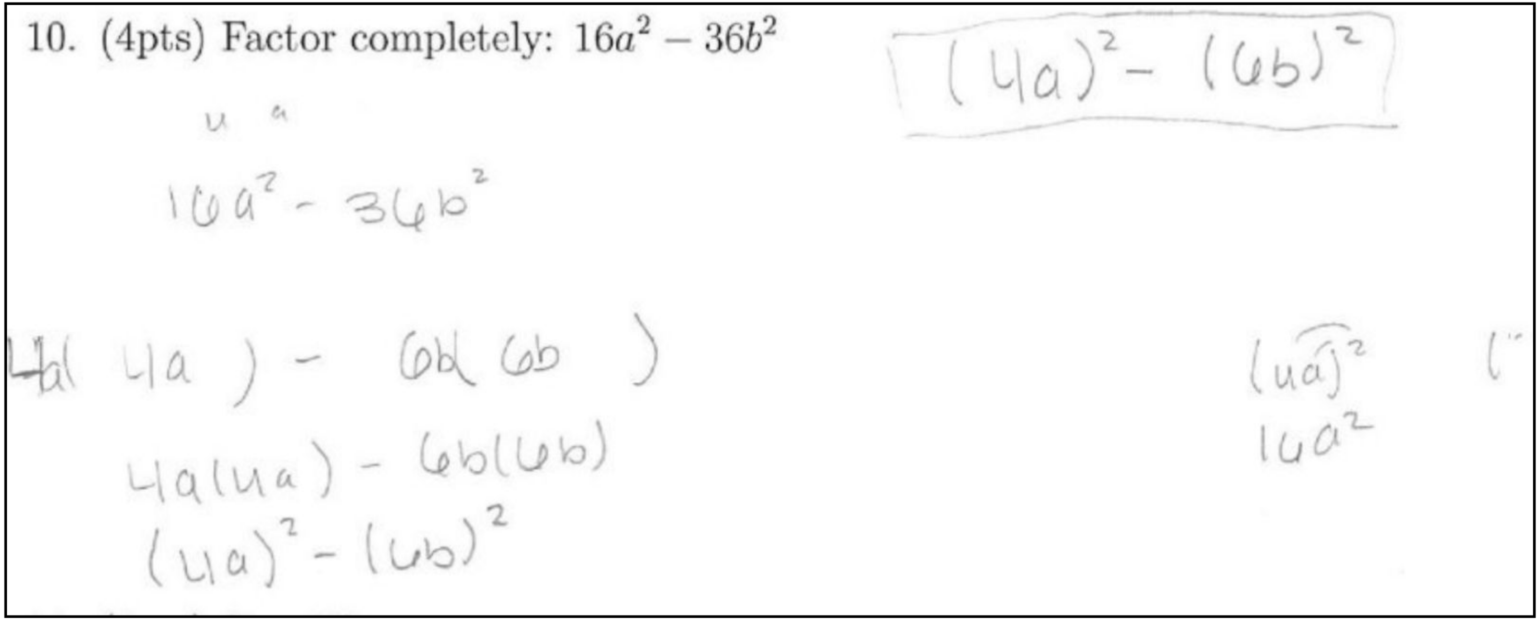
[Dialogue between interviewer and student]

Student: Oh man. It’s a **simple mistake. I just forgot how to do that particular question**.

The following slides have student mistakes.

In what phase is each mistake occurring?

Orienting
Planning
Executing
Checking



10. (4pts) Factor completely: $16a^2 - 36b^2$

$(4a)^2 - (6b)^2$

$16a^2 - 36b^2$

$4a(4a) - 6b(6b)$

$4a(4a) - 6b(6b)$

$(4a)^2 - (6b)^2$

$(4a)^2$

$16a^2$

Student AY: *Probably not simple. I don't know factoring at all, honestly.*

Orienting

Unsuccessful orientation:
Student is unable to identify one or more of initial state, goal state, and problem type.

8. (4pts) Use grouping to factor: $8x^2y - 2xy + 12xy - 3y$

$$8x^2y - 2xy + 12xy - 3y$$

$$2xy(4x-1) + 3y(4x-1)$$

$$(2xy+3y)(4x-1)$$

Unsuccessful execution:
Student is unable to completely carry out the process for solving the problem.

Student MF: *yeah. I um sometimes, or most of the time the majority of the problems I work on, it doesn't involve me factoring the y out, just the coefficient or the constant, and with that I guess that didn't make me realize I could have simplified it a little bit more, and then seeing that I could have taken out a y. Makes it a simple mistake.*

Executing

Checking was only applied when a student mentioned "checking" their work.

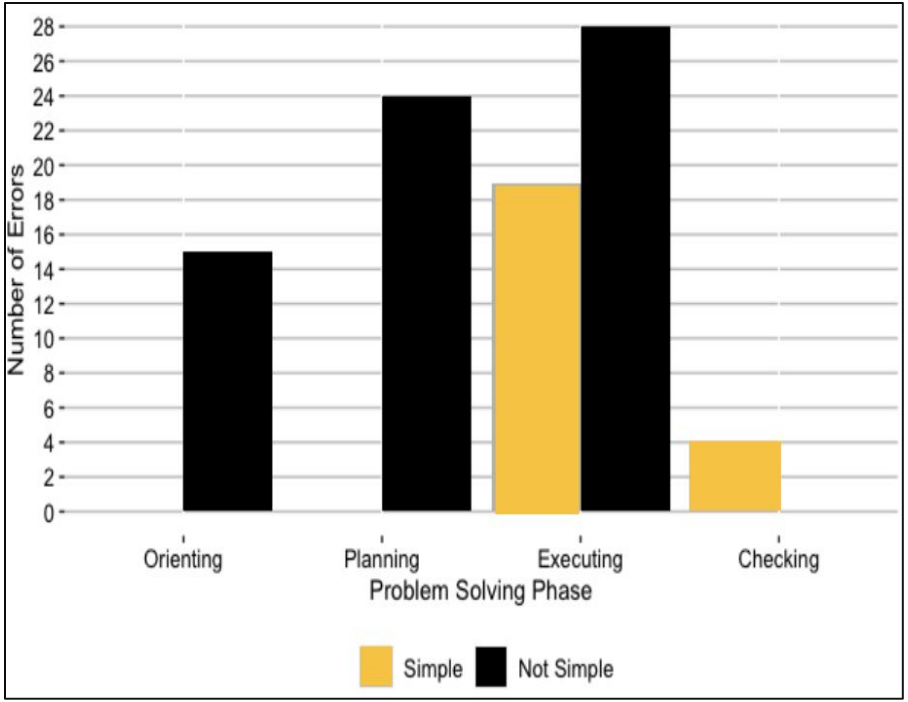
Checking mistakes can be difficult to distinguish from Executing mistakes. Checking did not come up often, and was only applied if a student discussed that they did indeed check their solution.

For example, when asked to completely factor $8x^2y-2xy+12xy-3y$, both Student J and Student MF had the final answer of $(2xy+3y)(4x-1)$.

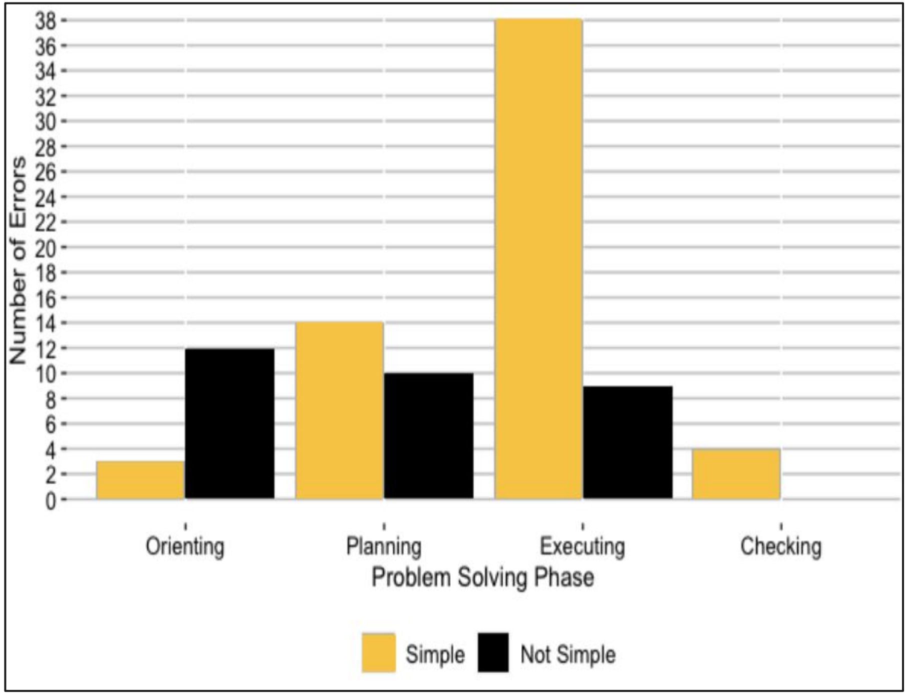
Student MF did not recognize that they could keep factoring (Executing), while Student J stated that they checked their work and thought that this final answer would be acceptable (Checking).

Overall results:

Classification of Mistakes by Researchers & Students



RESEARCH TEAM



STUDENTS

Discussion: Orienting Mistakes

- Orienting is the first phase in the problem-solving process, so if a student is stuck here: 1) they may not have much to write down or 2) they take a guess at how to solve the problem.
- Students recognized when they had orienting issues and often characterized these mistakes as being not simple, and thus often aligned with the Research Team's categorization.
- Statements such as "I kind of just like didn't know what was going on" and "Um I really I didn't know it" were made by students who identified orienting mistakes as not simple.

Discussion: Planning Mistakes

- Generally speaking, students who categorized their planning mistakes as being simple attributed them as resulting from memorization issues or stated something to the effect of ***"Now that I've seen it, I know how to do it."***
- When students were unsure of what strategy to use, they categorized their mistakes as not simple.

Discussion: Executing Mistakes

Research Team

Simple: 40%

Not Simple: 60%

Simple issues: typos, accidental, skill correctly demonstrated on other problem.

Not simple issues: Significant misunderstandings or lack of understanding of concepts being assessed on the problems.

Students

Simple: 81%

Not Simple: 19%

- Should have studied the problem/topic more.
- Have seen, have done, or knew the problem/topic before the exam.
- Had the right idea/approach to the problem.
- Made the problem harder; there was a simpler way.
- Rushed through the problem.
- Memorization issue.

Student (Course Grade)	Orienting	Planning	Executing	Checking
Student R (A):				
5 mistakes				
Simple	0	0	3	1
Not Simple	1	0	0	0

The Case of Student R:

- Student R's categorization of simple and not simple mistakes aligned well with the research team.
- Studied on a regular basis and assumed responsibility for mistakes.
- Recognized areas in which they had to spend more time or "to think about it." ← **More metacognitive than other students.**
- Student R is persistent: They worked "for like 30 minutes" on one exam problem until satisfied with their answer.

Student (Course Grade)	Orienting	Planning	Executing	Checking
Student C (D):				
15 mistakes				
Simple	0	5	5	0
Not Simple	2	3	0	0

The Case of Student C:

- Student C's definition of simple and not simple did not align well with researchers.
- Practice = Understanding → **Not simple mistakes are a result of NOT engaging with problem type enough**.
- Attributes making mistakes to not knowing how to study.
- Low self-efficacy and believed that their lack of ability to solve certain problems was beyond their control.

Student (Course Grade)	Orienting	Planning	Executing	Checking
Student RG (B):				
8 mistakes				
Simple	2	0	5	1
Not Simple	0	0	0	0

The Case of Student RG:

- All of their mistakes were identified as being simple.
- But had repeated mistakes ← **indicated lack of understanding, and thus not simple to research team.**
- Believes that simple mistakes are controllable (preventable) with little effort, so they are confident in their ability to prevent these mistakes in the future.
- Lack of awareness of what they know and do not know.

S: After you did it all did you go back and you check anything?

RG: No. I didn't check anything. Just turned it in.

S: Did you have time?

RG: I did. I had time but there was a football game on. I had to get out the door. I had to go. Rams were playin'.

Final Thoughts:

- Many students perceive their mistakes differently from us.
- Students who regularly engage in Self-Regulated Learning practices, such as self-reflection (e.g., Student R), may have closer alignment with us.
- Current work: What are tutors' perceptions of mistakes?
- Consider:
  - How do we encourage students to deeply examine their errors? (see the work by Heinze and collaborators)
  - How can we shift our practice to begin to incorporate the process of error analysis? (*It is not enough to just review an exam and correctly work problems.*)

Revisited: Why do we care?

- This allows us to better understand how students are identifying these mistakes and how it may differ from our perspectives.
- As a result, the help and support we offer may be misaligned with their needs. For example, we may make recommendations about test taking strategies that may not be suited for addressing their needs.
- Knowing students perceptions of their mistakes helps us better understand how we might help them.

Thank you!

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