HW 1 Selected Solutions

Prof. Shahed Sharif

1.4 These are

(a) $2e^{i\pi/2}$

(b) $\sqrt{2}e^{i\pi/4}$

(c) $2\sqrt{3}e^{i5\pi/6}$

(d) $e^{3\pi i/2}$

(e) $5e^{i\theta}$ where $\cos\theta = 3/5$ and $0 < \theta < -\pi/2$

(f) $5e^0$

(g) $\sqrt{6}e^{i\varphi}$ where $\cos\varphi = \frac{\sqrt{5}}{\sqrt{6}}$ and $0 < \varphi < -\pi/2$

(h) $\frac{4}{9}e^{i\pi}$ (it is easiest to convert to polar before exponentiating)

1.13 For (a), let $z = x + iy$. Setting $z = \bar{z}$, we get $x + iy = x - iy$, so $2iy = 0$, and hence $y = 0$. It follows that $z \in \mathbb{R}$. The converse results from reversing the argument.

For (b), if $z^2 = \bar{z}^2$, by part (a), $z^2 \in \mathbb{R}$. If $z^2 \geq 0$, then there are exactly two real possibilities for z. If $z^2 = -r$ for $r > 0$, then $z = \pm i\sqrt{r}$ where $\sqrt{r}$ refers to the positive square root of r (as usual).

The converse is omitted.

1.17 Rather than use the hint, it's easier just to use the Math 470 result that a degree n polynomial over a field has at most n roots. We apply this to the polynomial $p(z) = z^n - 1$. For $m \in \mathbb{Z}$, we have

$$\begin{aligned} p(e^{2\pi i m/n}) &= (e^{2\pi i m/n})^n - 1 \\ &= (e^{2\pi i})^m - 1 \\ &= 1^m - 1 \\ &= 0. \end{aligned}$$

Notice that as m varies in $0 \leq m \leq n-1$, the numbers $e^{2\pi m/n}$ have distinct arguments in $[0, 2\pi)$, and hence are distinct complex roots. This yields n distinct roots for p, so there are no additional roots.