

Homework 1

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- 1.2.4 They are not row-equivalent, since they do not have the same solution set;

$$(1 - i, 2, 3, 0)$$

is a solution to the first system, but not to the second. (Calculation omitted.)

That's really all you have to do. But you may ask how I came up with this! Well, I wrote down the two associated matrices,

$$\begin{bmatrix} 2 & -1+i & 0 & 4 \\ 0 & 3 & -2 & 5 \end{bmatrix} \text{ and } \begin{bmatrix} 1+\frac{i}{2} & 8 & -i & -1 \\ \frac{2}{3} & -\frac{1}{2} & 1 & 7 \end{bmatrix}.$$

Then I computed the rrefs with a computer, obtaining respectively

$$\begin{bmatrix} 1 & 0 & \frac{1}{3}i - \frac{1}{3} & -\frac{5}{6}i + \frac{17}{6} \\ 0 & 1 & -\frac{2}{3} & \frac{5}{3} \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 0 & -\frac{708}{4909}i + \frac{6702}{4909} & -\frac{1998}{4909}i + \frac{46620}{4909} \\ 0 & 1 & -\frac{944}{4909}i - \frac{882}{4909} & -\frac{2664}{4909}i - \frac{6566}{4909} \end{bmatrix}.$$

These aren't the same, so one has different solution sets. I randomly picked a nontrivial solution to the first system, which with probability 1 will not be a solution to the second. If I'm **really** unlucky, I'll have to try again; but as I said, the bad luck has probability zero.

- 1.2.6 The easiest method is to prove the contrapositive by use of the rref. (One can do it directly too, but the proof is essentially the same idea, just messier.) If one system has more equations than the other, then add $0 = 0$ to the other system enough times that they have the same number of equations. Put each system into rref. Note that there are at most two pivots, so all but two of the rows are zero rows. As a result, we will only consider the top two rows of the rrefs.

If the systems are not equivalent, then the rref's must not be the same. For a two-column system, there are four types of rref's, with top two rows one of the following:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & a \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

This is easy to see, since there are at most two pivots. I have enumerated all possible ways to get two pivots. In the above a can be any element of the base field; two different a values give two different rref's.

Now it remains to enumerate solution sets. These are, respectively,

$$\{(0, 0)\}, \{(at, -t) \mid t \in F\}, \{(t, 0) \mid t \in F\}, \{(s, t) \mid s, t \in F\}.$$

Notice that both $(1, 0)$ and $(0, 1)$ are both in the last set only, $(1, 0)$ is in the last two sets only, and $(a, -1)$ is in the second and last set only.

Thus for any pair of distinct solution sets, there is a vector in one of the sets that is not in the other. This holds even for the second set, if we take two different values of a ; say a, a' , for then $(a, -1)$ is not in the set $\{(a't, -t) \mid t \in F\}$. Thus the solution set characterizes the rref. Since we know our two systems must have different rrefs, we see that their solutions sets are different.

1.3.1 The associated matrix is

$$\begin{bmatrix} 1-i & -i \\ 2 & 1-i \end{bmatrix}.$$

Next we row-reduce. I'll first multiply the top row by $1+i$ to get

$$\begin{bmatrix} 2 & -i+1 \\ 2 & 1-i \end{bmatrix}.$$

Subtracting the top row from the bottom yields

$$\begin{bmatrix} 2 & 1-i \\ 0 & 0 \end{bmatrix}.$$

Thus the solution set is

$$\left\{ \left(\frac{i-1}{2}t, t \right) : t \in \mathbb{C} \right\}.$$