Presentation Topics

You are expected to give a 20 minute talk on a topic of your choice that is related to algebraic geometry. The talk should be mathematically substantive, but not overly technical. The ideal is to give the main ideas of a theorem or body of work without getting bogged down in details. I recommend a goal of just one calculation or proof. Of course, being able to boil down a complex topic into a short talk assumes a depth of understanding. I should also warn you that I ask lots of questions.

At least two weeks prior to the date of your talk, you must hand in a 1 page summary so that I can give feedback. If you like, you may provide this summary to the audience, though this isn't necessary.

Dates. You must choose a topic by **October 31**. The topic must be specific; you can't just say, "algebraic groups". Talks will be given starting **November 19** until the last day of class.

Topics. Some of the topics below are broad, so you'll have to narrow your focus. Wikipedia is a good place to start, and often gives a list of subtopics. You can also come up with your own topic, as long as I approve it ahead of time. They are ordered *very roughly* from simplest to most advanced. Broad topics of course are hard to categorize, since they contain both simple and challenging aspects.

1. Plücker coordinates. This is a way of specifying points in projective 3-space which is useful for computational applications.
2. Resultants. Resultants are a computational tool, based on determinants of matrices, with surprisingly powerful applications.
3. Elliptic curves. Elliptic curves are the most important type of variety in number theory, and their importance in cryptography has given rise to theory and algorithms related to elliptic curves.
4. Hilbert function. Hilbert functions allow us to define the dimension of a variety.
5. Gröbner basis algorithms. We cover the simplest Gröbner basis algorithm, but there are other faster implementations. *Yoko, 12/5*
6. Applications. There are tons of applications. Some examples:
   - Robotics *Abigail, 12/3*
   - Bezier curves *Waleed, 12/5*
   - Computer vision
   - Integer programming

- Error-correcting codes

7. Cap set problem. Related to combinatorics and the game SET, there has been huge progress in this area in recent years. *Ian, 12/3*

8. Gröbner basis efficiency. Unfortunately, Gröbner basis algorithms are very slow in worst-case situations. However, in practice for most examples the computation is very fast. What are these worst-case situations? Is it known how rare they are. *Noah, 12/5*

9. Rational varieties. The simplest varieties: those birational to $\mathbb{A}^n$. But how can you tell if something is a rational variety?

10. Algebraic groups. They are both varieties and groups! There is a vast literature on these objects. *Jesse, 12/3*

11. Enumerative geometry. The general problem of counting things in algebraic geometry: the number of lines with various properties, for instance. Pick a problem and describe methods for approaching it.

12. Tropical geometry. A weird kind of degenerate geometry that has surprising applications and relationships with algebraic geometry.

13. Algebraic surfaces. There are lots of types of algebraic surfaces: cubic surfaces, del Pezzo surfaces, K3 surfaces, Enriques surfaces, and many more. Each type has interesting properties. Pick one and talk about something special about it.

14. Finite fields. In computer science and number theory, algebraic geometry is most often conducted over finite fields. How are these constructed, and how do we do computations over them?

15. Chow forms. These are multidimensional generalizations of Plücker coordinates.

16. Genus. The classification of curves begins with the study of genus. What is the genus, and how do we compute it?

17. Real algebraic geometry. Algebraic geometry over the reals is quite a bit trickier than over either $\mathbb{Q}$ or $\mathbb{C}$. I don't know any methods myself, so it would be interesting to see what is known.

18. Invariant theory. Given a set of varieties, we typically want to classify them by identifying "invariants"; that is, numbers which don't change under some notion of isomorphism. There is a huge machinery of invariant theory that has been developed to tackle this problem; the genus (above) is just one example.

19. Differential forms. This is the analog of differential calculus in algebraic geometry.

20. ______________________________. Pick your own topic! Make sure to run it by me first.