HW 7

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HW 7: §2.6/1, 3, 6, 10; §2.7/2, 7; §2.8/1, 3

2.6.3 Let $f \in I$ nonzero, and let $m = \mathrm{LT}(f)$. Since $[f]_G = 0$, m does not go in the remainder column in the division algorithm, and hence $\mathrm{LT}(g) \mid m$ for some $g \in G$. This implies that $\mathrm{LT}(f) \in \langle \mathrm{LT}(G) \rangle$. As this holds for all $f \in I$, the G is a Gröbner basis for I.

2.6.6 It certainly does, since the S-polynomial makes reference to the leading term. Take $f = x + y^2$ and $g = x - y^2$. With respect to lex, the S-polynomial is

$$f - g = 2y^2.$$

With respect to grlex, the S-polynomial is

$$f + g = 2x.$$

2.7.2 I will do lex only. For (a), we get

$$\langle x^2y - 1, xy^2 - x, x^2 - y, y^2 - 1 \rangle .$$

For (b), we get -3 as our first S-polynomial, hence this is the unit ideal and $V(I)$ is empty.

For (c), the S-polynomial is 0, so we already have a Gröbner basis.