Worksheet 22

1. In $\mathbb{Z}/31\mathbb{Z}[x]$, compute the remainder when we divide $x^5 + 23x^4 - 27x^3 + 20x^2 + 10x - 15$ by $x - 4$.
2. Let $p(x) = x^2 - 1$ and $f(x) = x^{23} + 278x^{17} - 153x^{10} + x^3 - 12$.
   (a) Show, in $\mathbb{R}[x]$, that $p(x)$ does not divide $f(x)$.
   (b) Show, in $\mathbb{Z}/5\mathbb{Z}[x]$, that $p(x)$ does divide $f(x)$.
3. In $\mathbb{Q}$, let R be the smallest subring containing 1 and $\frac{1}{2}$, and let H be the smallest subgroup containing 1 and $\frac{1}{2}$. Does $R = H$?