Worksheet 20

1. Show that the function

$$\varphi : \mathrm{GL}_2(\mathbb{R}) / \mathrm{SL}_2(\mathbb{R}) \to \mathbb{R} \setminus \{0\}$$

   given by $\varphi([A]) = \det A$ is well-defined.

2. Show that the φ from above is a bijection.

3. Show that $A_n \lhd S_n$, and show that

$$S_n / A_n \cong \mathbb{Z} / 2\mathbb{Z}.$$

   (Remember to show that your constructions are well-defined.)