Worksheet 17

For each function below, show that it is a homomorphism. Then compute the kernel and image.

1. $\det : GL_2(\mathbb{R}) \to \mathbb{R}\backslash\{0\}$ (the determinant)
2. $\pi : \mathbb{Z}/12\mathbb{Z} \to \mathbb{Z}/3\mathbb{Z}$ given by $[x] \mapsto [x]$. Also show π is well-defined.
3. $\varphi : \mathbb{R}\backslash\{0\} \to \mathbb{R} - \{0\}$ given by $x \mapsto x^2$.
4. $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = \left(\frac{x+y}{2}, \frac{x+y}{2}\right)$.