Worksheet 10

Recall the following elements of the braid group on three strands: σ_1 takes the first "tail" over the second, and σ_2 takes the second tail over the third. We say we *apply* an element of the braid group (say, $\sigma_1\sigma_2\sigma_1^{-1}\sigma_2^3$) to a pattern of strands if we do out the operation, going right to left. The starting pattern of strands (completely unentangled) is called the *trivial braid*. If we apply an element α of the braid group to the trivial braid, we call the resulting pattern of strands the *braid* associated to α.

1. What is the order of σ_1?
2. Find the braid associated to $\alpha = \sigma_1\sigma_2^{-1}\sigma_1^2\sigma_2\sigma_1^{-1}$.
3. Find the braid associated to α^{-1}.
4. Consider the element of the braid group taking the first tail over both the other two tails, ending up on the far right. Can you write this element in terms of σ_1 and σ_2?
5. Same question, except we want to take the third tail under the other two tails, ending up on the far left.
6. You are given a product α of σ_is and σ_i^{-1}s. Give a method for determining if the the *tails* of the braid associated to α are in the same order as you started with. (So the tops and bottoms of the braid should be the same sequence of colors.)
7. Find a nontrivial equality of elements of the braid group. Two elements of the braid group are equal if they give you the same braid. (For example, $\sigma_1\sigma_1^{-1} = \sigma_2\sigma_2^{-1}$ is an equality, but it is trivial because it holds in *any* group, not just the braid group.)