Worksheet §0

1. Each of the following statements is false. Find a counterexample. Make sure to justify that your counterexample is, in fact, a counterexample!

   (a) If a function is injective, then it is surjective.

   (b) If a function is surjective, then it is injective.

   (c) For any sets A and B, suppose $f : A \to B$ and $g : B \to A$ are functions for which $f \circ g = \mathrm{id}_B$. Then f is a bijection.

2. Prove that $\forall k \geq 0$,

$$\frac{1}{2^1} + \frac{1}{2^2} + \cdots + \frac{1}{2^k} = 1 - \frac{1}{2^{k+1}}.$$

3. Define $T(n)$ for $n \geq 0$ by

$$T(n) = \begin{cases} 1 & \text{if } n = 0 \\ \displaystyle\sum_{k=0}^{n-1} T(k) & \text{if } n \geq 1 \end{cases}$$

   Find an explicit formula for $T(n)$, and prove it.