HW 9 Due: Thursday, March 27

Do 2/53, 55, and the following.

- A. Prove that if p is a prime and $a \in \mathbb{Z}$ with gcd(a,p) = 1, then $a^{p-1} \equiv 1 \pmod{p}$. (Hint: use Lagrange's Theorem in U(p).)
- B. Determine if 2 is a generator of U(47).
- C. Prove that if G is a cyclic group, then every subgroup is cyclic. (Hint: take $H < G = \langle g \rangle$, and pick $h \in H$ such that $h = g^n$ with smallest positive n.)
- D. Show that every subgroup of \mathbb{Z} is of the form $n\mathbb{Z}$ for some n.
- E. Suppose H_1, H_2 are subgroups of a group G, and the orders of H_1, H_2 are relatively prime integers. Prove that $H_1 \cap H_2$ is the trivial subgroup.
- F. In $GL_2(\mathbb{R})$, let $H = SL_2(\mathbb{R})$, the determinant 1 matrices. Determine what the equivalence classes are for $\equiv \pmod{H}$.