Name:

Math 470: Exam 5

Make sure to show all your work as clearly as possible. This includes justifying your answers if required. Calculators are not allowed.

You may use any result from the chapters covered in the text or from lecture. You may not use the results of homework or worksheet problems.

1. (10 pts) In S_5, let $H = \langle (1\,3)(2\,4\,5) \rangle$. List 3 different elements which are congruent to $(1\,2)$ mod H.

Solution: We have

$$H = \{(1), (1\,3)(2\,4\,5), (2\,5\,4), (1\,3), (2\,4\,5), (1\,3)(2\,5\,4)\}.$$

Multiplying everything by $(1\,2)$ on the left, we obtain

$$(1\,2)H = \{(1\,2), (1\,3\,2\,4\,5), (1\,2\,5\,4), (1\,3\,2), (1\,2\,4\,5), (1\,3\,2\,5\,4)\}.$$

Now choose any 3.

2. (20 pts) Without using any facts about cosets except their definition, prove the following fact: if G is a group, H a subgroup, and $a, b \in G$ satisfy $b \in aH$, then $bH \subseteq aH$.

Solution: Suppose $b \in aH$. This means $\exists h \in H$ such that $b = ah$. Let $c \in bH$. This means $\exists h' \in H$ such that $c = bh'$. Then

$$\begin{aligned} c &= bh' \\ &= ahh'. \end{aligned}$$

But H is a subgroup, so is closed under multipliction. Thus $hh' \in H$; write $h'' = hh'$. Then $c = ah''$, and hence $c \in aH$. Since c was an arbitrary element of bH, we have $bH \subseteq aH$.

3. (20 pts) In D_{120}, let $H = \langle \sigma^{12} \rangle$. Compute the index $[D_{120} : H]$ (number of left cosets). Prove your answer.

Solution: First, I claim the order of σ^{12} is 5. For

$$(\sigma^{12})^5 = \sigma^{60} = e,$$

so 5 is a period. Since the order divides every period, the order is 1 or 5. As $\sigma^{12} \neq e$, the order is 5. It follows that $\#H = 5$. We have $\#D_{120} = 120$. Thus the index is $120/5 = 24$.