Name:

**Math 470: Exam 4**

Make sure to show all your work as clearly as possible. This includes justifying your answers if required. Calculators are not allowed.

You may use any result from the chapters covered in the text or from lecture. You may not use the results of homework or worksheet problems.

1. (10 pts) Give an example of a subgroup of $\mathbb{R} - \{0\}$ which has exactly two elements. (The operation is multiplication.) You do not have to prove your answer.

> **Solution:** There's only one: $\{1, -1\}$.

2. (20 pts) Prove that the only finite subgroup of $\mathbb{R}$ is $\{0\}$.

> **Solution:** Let $H < \mathbb{R}$ be a nontrivial subgroup. This implies that $\exists h \in H$ with $h \neq 0$. For $n \in \mathbb{Z}$, if $nh = 0$, then by the Zero Product Property (or whatever you call it), $n = 0$ or $h = 0$; by hypothesis $h \neq 0$, so $n = 0$. In particular, $h$ has infinite order, and hence $\langle h \rangle$ is infinite. As $\langle h \rangle \subset H$, $H$ must also be infinite.

3. (20 pts) In $D_{80}$, compute the order of $\sigma^{12}$. Prove your answer.

> **Solution:** I claim that the order is 10. We have
>
> $$\begin{aligned}(\sigma^{12})^{10} &= \sigma^{120} \\ &= (\sigma^{40})^3 \\ &= e^3 = e.\end{aligned}$$
>
> Thus 10 is a period. As the order divides any period, the order is $1, 2, 5$, or 10. Certainly $(\sigma^{12})^1 \neq e$. We have $(\sigma^{12})^2 = \sigma^{24}$, and since $\sigma$ has order 40, $\sigma^{24} \neq e$. Lastly,
>
> $$(\sigma^{12})^5 = \sigma^{60} = \sigma^{40}\sigma^{20} = \sigma^{20} \neq e.$$
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> Thus we have eliminated $1, 2, 5$ as orders, so the order is 10.