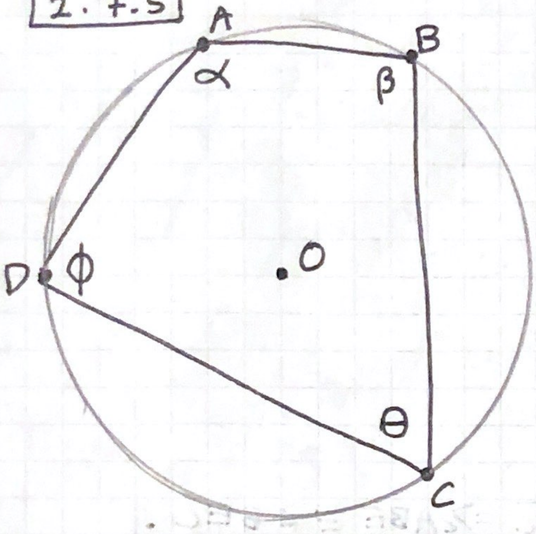
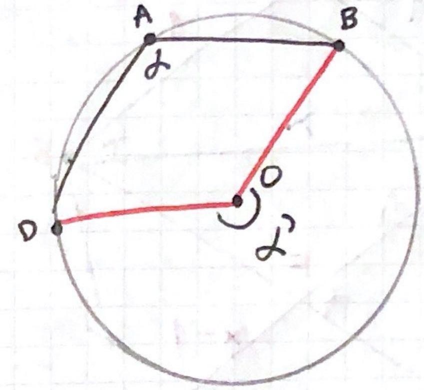
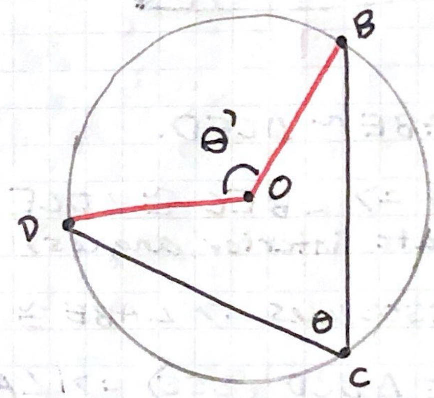
410 HW 4: $\boxed{2.7.5,\ 2.8.1,\ 3.2.5}$

$\boxed{2.7.5}$







WTS: $\alpha + \theta = \pi$, $\beta + \phi = \pi$.

Given quadrilateral (inscribed) ABCD, construct $\overline{OB}$, $\overline{OD}$.

Theorem – central angle is twice the inscribed angle.

$\Rightarrow \alpha' = 2\alpha$ and $\theta' = 2\theta$.

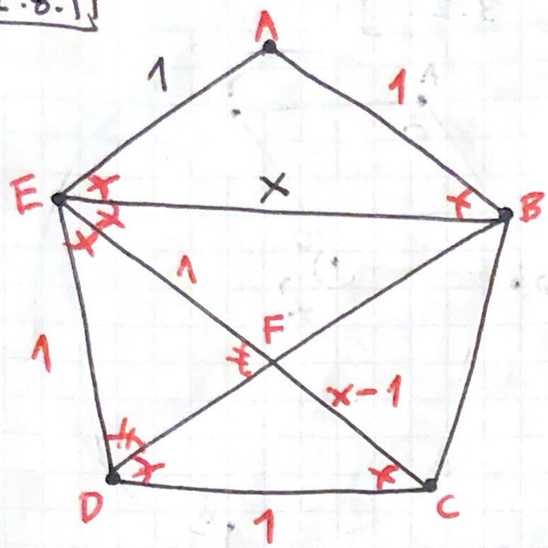
see $\alpha' + \theta' = 2\pi$

Substituting, $2\alpha + 2\theta = 2\pi$

$\alpha + \theta = \pi$ as desired.

(construct $\overline{OA}$, $\overline{OC}$ and follow same logic to show $\beta + \phi = \pi$).

$\boxed{2.8.1}$



<u>WTS</u> $\Delta ABE \sim \Delta CFD$.

$\overline{BE} \parallel \overline{CD} \Rightarrow \angle BEC \cong \angle DCE$. $\overline{AB} \parallel \overline{EC} \Rightarrow \angle ABE \cong \angle BEC$.
(alternate interior angles)

ΔABE isosceles $\Rightarrow \angle ABE \cong \angle \angle AEB$

$\Delta ABE \cong \Delta BCD$ (SSS) $\Rightarrow \angle AEB \cong \angle BDC$

$\Delta ABE \sim \Delta CFD$ (AA).

<u>WTS</u> $\overline{FC} = x-1$ by showing ΔEFD isosceles.

$\Delta ABE \cong \Delta DEC$ (SSS) $\Rightarrow \angle DEC \cong \angle ABE$

call $\angle FDC$ θ, then $\angle CFD = \pi - 2\theta$
then $\angle EFD = \pi - (\pi - 2\theta) = 2\theta$
see $\angle AED = 3\theta$ and $\angle AED \cong \angle EDC$
$\angle EDC = \angle EDF + \theta \Rightarrow \angle EDF = 2\theta$
so $\angle EFD \cong \angle EDF \Rightarrow \Delta EFD$ isosceles $\Rightarrow \overline{ED} \cong \overline{EF} \Rightarrow \overline{EF} = 1$

$\overline{FC} = \overline{EC} - \overline{EF} = x - 1$

$$\Delta ABE \sim \Delta CFD \Rightarrow \frac{\overline{EB}}{\overline{DC}} = \frac{\overline{AB}}{\overline{FC}}$$

substituting, $$\frac{x}{1} = \frac{1}{x-1}$$

$x^2 - x - 1 = 0$ as desired.

$\boxed{3.2.5}$ Given: a line $\mathcal{L}$ and a point P <u>not on</u> $\mathcal{L}$.

- let $\mathcal{L}$ be represented as $y = ax + c$
  let $P = (x_0, y_0)$
  let $M \parallel \mathcal{L}$, M represented as $y = ax + c'$ $(c' \neq c)$
  let $P = (x_0, y_0)$ lie on M.
  so we have $y_0 = a x_0 + c'$

Now - assume (for contradiction) $\exists$ M' through P such that M' represented as $y = ax + c''$ with $c'' \neq c'$ (i.e., M' is distinct from M, but parallel to $\mathcal{L}$ and through P).
we have $y_0 = a x_0 + c''$

But - $y_0 = ax_0 + c'$ and $y_0 = ax_0 + c''$
$ax_0 + c' = ax_0 + c''$ $\Rightarrow$ $c' = c''$ $\}$ contradiction
$\Rightarrow$ $M' = M$, i.e. the parallel to $\mathcal{L}$ through P is <u>unique</u>.