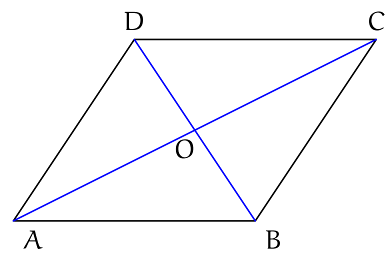
HW 3
Due: Tuesday, February 18

For constructions, do out an example construction. Label the points and refer to them in both the definition of the construction and the proof (as we've done in class).

Do problems 1.4.2, 1.4.3, 1.4.4, 2.1.1, 2.2.1, 2.2.2, and the following:

2.2.1 Consider the parallelogram below.



We showed that $AD = BC$, and also that $\angle ADO \cong \angle CBO$ and $\angle DAO \cong \angle BCO$. By ASA, $\triangle ADO \cong \triangle CBO$. In particular, $DO = BO$ and $AO = CO$. The claim follows.

A. Given $\angle BAC$ and point P inside the angle, construct a line ℓ through P so that if ℓ intersects $\overrightarrow{AB}$ at D and $\overrightarrow{AC}$ at E, then $\angle ADE \cong \angle AED$.

B. Given a square, construct a square with twice the area.

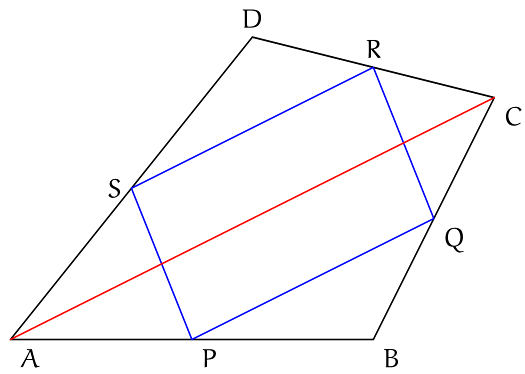
Construct a diagonal of the square, and then a square on the diagonal. If a side of the original square has length s, then the diagonal has length $s\sqrt{2}$. The ratios of the areas is then $(s\sqrt{2})^2/s^2 = 2$.

C. Given a square, construct a square with three times the area.

D. Given a square and positive integer n, describe a method for constructing a square with n times the area. You do not have to prove your answer.

E. Show that if we take any *convex* quadrilateral and connect the midpoints of the 4 sides, we obtain a parallelogram. (Convex means all the vertices point outwards.)

Consider quadrilateral $ABCD$ and the derived quadrilateral $PQRS$, drawn below (that is, P, Q, R, S are the midpoints of the indicated sides). We wish to show that $PQRS$ is a parallelogram.



Comparing $\triangle DRS$ to $\triangle DCA$, we see that $DR/RC = DS/SA = 1$ since R, S are midpoints of the respective sides. By the converse to Thales' Theorem, $\overline{RS}$ is parallel to $\overline{AC}$. By similar reasoning, $\overline{PQ}$ is parallel to $\overline{AC}$. It follows that $\overline{RS}$ is parallel to $\overline{PQ}$.

A similar argument holds for $\overline{PS}$ and $\overline{QR}$. The claim follows.