HW 2
Due: Tuesday, February 11

For constructions, do out an example construction. Label the points and refer to them in both the definition of the construction and the proof (as we've done in class).

Do problems 1.2.1, 1.3.3, 1.4.1, and the following:

1.3.3 Note that for 1.3.3, *on a given line segment* means that the segment should be a side of the resulting square. Since the wording was unclear to some people, I decided not to grade this problem.

1.4.1 We will prove the contrapositive. Assume that $\frac{AP}{PB} = \frac{AR}{RC}$, and we will prove that $\overline{AR}$ is parallel to $\overline{BC}$. Construct a line ℓ through P parallel to $\overline{BC}$. Let Q be the intersection of ℓ and $\overline{AC}$ as in the picture in the text. By Thales' Theorem, $\frac{AP}{PB} = \frac{AQ}{QC}$. But using our hypothesis we get

$$\frac{AQ}{QC} = \frac{AR}{RC}.$$

Next we show that $AQ = AR$ using algebra. Once we have that, we must have $Q = R$, and hence ℓ is the same as the line through P and R (by the uniqueness of lines through two points), and hence the latter line is parallel to $\overline{BC}$ as required.

Okay, now for the algebra! Let $AQ = x$, $AR = y$, and $AC = c$. Our equality of fractions above looks like

$$\frac{x}{c-x} = \frac{y}{c-y}.$$

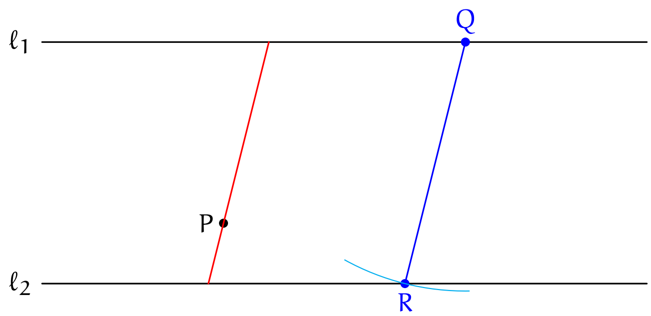
Hence $cx - xy = cy - xy$, or $cx = cy$. Therefore $x = y$.

A. Given two segments $\overline{AB}$ and $\overline{BC}$, show how to construct a right triangle with one side of length AB and hypotenuse of length BC.

B. Given three vertices of a parallelogram, show how to construct the fourth.

C. You are given two parallel lines ℓ_1 and ℓ_2 and a point P between them. You are also given a sufficiently long segment $\overline{AB}$. Show how to construct a line t through P so that if t intersects ℓ_1 at C and ℓ_2 and D, then $CD = AB$. (Hint: draw a parallelogram.) You do not have to prove your answer.

Consider the picture below. We start with only the black parts of the drawing, and a segment $\overline{AB}$ (not pictured).



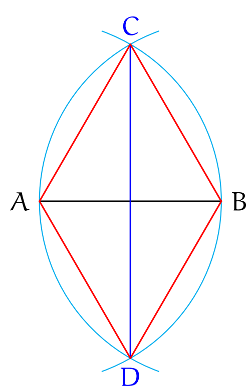
Our construction is as follows:

1. Choose any point on ℓ_1; I choose Q. Construct the circle with radius AB centered at Q.
2. Let R be any intersection of this circle with ℓ_2. (Note that AB being large means that such an intersection exists.) Construct $\overleftrightarrow{QR}$.
3. Construct a line t parallel to $\overleftrightarrow{QR}$ through P.

To show the construction is correct, observe that the quadrilateral enclosed by ℓ_1, ℓ_2, t, and $\overleftrightarrow{QR}$ is a parallelogram. We showed that opposite sides of a parallelogram are equal, and hence the desired portion of t has length equal to QR. But by construction, $QR = AB$. The claim follows.

D. Use SAS and/or ASA to prove that the construction of the perpendicular bisector of a segment is correct.

Recall the construction below.



Here, the cyan arcs are from the circles of radius AB centered at A and B respectively. The perpendicular bisector is supposed to be $\overleftrightarrow{CD}$. There are several methods to prove this; here is the one I outlined in class.

Let M be the central point in the picture, not labelled. (That is, the intersection of the two lines.) By contruction, all red segments are equal; that is, $AC = BC = AD = BD$. By Pons Asinorum, $\angle CAB \cong \angle CBA$. Similarly $\angle DAB \cong \angle DBA$. It follows that $\angle CAD \cong \angle CBD$, and hence by SAS, $\triangle CAD \cong \triangle CBD$. This implies that $\angle ACM \cong \angle BCM$, and so by SAS, $\triangle ACM \cong \triangle BCM$. Therefore $AM = BM$ and $\angle AMC \cong \angle BMC$. These last two angles are supplementary, and since they are congruent, they must be right angles. The claim follows.

E. Prove that if parallel lines are cut by a transversal, then corresponding angles are equal. (Hint: use the result that if corresponding angles are equal, then the lines are parallel; and also use the Parallel Postulate.)