Worksheet 3
Due: Thursday, February 5

Put full names of the group at the top. Pick one person to record who hasn't before. Star their name.

1. In $\mathbb{F}^3$, let $V_1 = \{(x, 0, 0) : x \in \mathbb{F}\}$, $V_2 = \{(y, y, 0) : y \in \mathbb{F}\}$, and $W = \{(u, v, 0) : u, v \in \mathbb{F}\}$. Show that

$$V_1 + V_2 = W.$$

2. Let V be a vector space and $V_1, V_2 \subseteq V$ be subspaces. Define $V_1 \widetilde{+} V_2$ as

$$\begin{aligned} V_1 \widetilde{+} V_2 = \{v \in V : \exists m, n \in \mathbb{N}, v_1, \ldots, v_m \in V_1, w_1, \ldots, w_n \in V_2, \\ \text{with } v = v_1 + \cdots + v_m + w_1 + \cdots + w_n\}. \end{aligned}$$

What is the difference between $V_1 + V_2$ and $V_1 \widetilde{+} V_2$?

3. Using your intuitive notion of dimension, how would you relate the dimensions of V_1, V_2, and $V_1 + V_2$? I'm looking for a precise algebraic relationship, but no justification is necessary.