Homework 6 Selected Solutions
Due: Tuesday, March 3

2B.3a Take

$$v_1 = (3,1,0,0,0),\ v_2 = (0,0,7,1,0), \text{ and } v_3 = (0,0,0,0,1).$$

Let $B = (v_1, v_2, v_3)$. I claim that B is a basis for U. For if

$$a_1 v_1 + a_2 v_2 + a_3 v_3 = 0,$$

then simplifying the left side we get

$$(3a_1, a_1, 7a_2, a_2, a_3) = (0,0,0,0,0).$$

It follows that $a_1 = a_2 = a_3 = 0$.

For spans, if $v = (x_1, x_2, x_3, x_4, x_5) \in U$, then by definition of U,

$$v = (3x_2, x_2, 7x_4, x_4, x_5) = x_2 v_1 + x_4 v_2 + x_5 v_3.$$

Thus $v \in \text{Span}(B)$.

2B.6 This is false: take

$$B = (x^3, x^3 + x^2, x^3 + x^2 + x, x^3 + x^2 + x + 1).$$

Certainly none of the elements of B has degree 2, so we just have to show that B is a basis. We already know that $(x^3, x^2, x, 1)$ is a basis. By exercise 2B.9, it follows that B is a basis.

2B.8 This is false. Take $V = \mathbb{R}^4$ and $U = \text{Span}(e_1, e_2, (0,0,1,1))$. Let $v_i = e_i$ for each i. Certainly (e_1, e_2, e_3, e_4) is a basis for V, and $e_1, e_2 \in U$. To have $e_3 \in U$, we'd need a solution to

$$ae_1 + be_2 + c(0,0,1,1) = e_3,$$

or

$$(a, b, c, c) = (0,0,1,0).$$

This is impossible: we'd need $c = 1$ and $c = 0$ at the same time. Similarly $e_4 \notin U$.