

Worksheet §2.6–2.10

1. Negate the following statements.
 - (a) If n is smooth, then n is not prime.
 - (b) Some cats are mammals.
 - (c) $\exists x \in \mathbb{R}$ such that $\forall y \in \mathbb{R}, xy < y$.
 - (d) $\forall \varepsilon > 0, \exists \delta > 0$ such that if $|x - a| < \delta$, then $|f(x) - f(a)| < \varepsilon$.
 - (e) If the kernel of φ is a subset of H , then $\exists \psi$ such that $\varphi = \pi \circ \psi$.
2. True or false?
 - (a) $\forall x \in \mathbb{N}, \exists y \in \mathbb{N}$ such that $x = y + 1$.
 - (b) $\exists x \in \mathbb{N}$ such that $\forall y \in \mathbb{N}, x = y + 1$.
 - (c) $\forall x \in \mathbb{Q}, \exists y \in \mathbb{Q}$ such that $x^2 + 10x - y = 0$.
 - (d) $\forall y \in \mathbb{Q}, \exists x \in \mathbb{Q}$ such that $x^2 + 10x - y = 0$.
 - (e) $\forall r \in \mathbb{R}, r > 0, \exists n \in \mathbb{N}$ such that $nr > 1$.
 - (f) $\exists A \subseteq \mathbb{N}$ such that $\forall n \in A, 2n \in A$ and $2n + 1 \notin A$.
 - (g) $\forall S \subseteq \mathbb{R}, \exists s \in S$ such that $\forall r \in S, s \leq r$.
 - (h) $\exists B \subseteq \mathbb{R}$ such that B is infinite and $\forall x, y \in B, |x - y| \geq 1$.