Worksheet §12.4

1. Suppose $f : X \to Y$ and $g : Y \to X$ are functions with $g \circ f = \mathrm{id}_X$. Give an example to show that f need not be bijective. Why doesn't this contradict our theorem on invertible functions?
2. Let $f : X \to Y$ and $g : Y \to Z$ be surjective functions. Prove that $g \circ f$ is surjective.
3. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $f(x, y) = (x + y, x - y)$.
   (a) Find an inverse function for f. (Hint: one method is to set $(u, v) = f(x, y)$ and solve for (x, y).)
   (b) Prove that your function for (a) is the inverse of f.
   (c) Prove that f is a bijection.
4. Suppose $f : X \to Y$ and $g : Y \to X$ are functions with $g \circ f = \mathrm{id}_X$. Show that f is injective and g is surjective.