Worksheet §11.5

1. Construct the addition and multiplication tables for $\mathbb{Z}_6$.

2. Let $A = \mathbb{Z} \times \mathbb{N}$. Construct a relation $\sim$ on A defined by

$$(a, b) \sim (c, d) \text{ if } ad = bc.$$

   (a) Show that $\sim$ is an equivalence relation.

   (b) Describe the set of equivalence classes.

   (c) Let B be the set of equivalence classes. Show that the operation $+$ on B defined by

$$[(a, b)] + [(c, d)] = [(a + c, b + d)]$$

   is *not* well-defined.

   (d) What is the correct definition of $+$?