Worksheet §11.3

1. Define an equivalence relation $\sim$ on $\mathbb{R}^2$ by $(x, y) \sim (u, v)$ if and only if $y = v$. Describe the partition of $\mathbb{R}^2$ resulting from this relation.

2. Define an equivalence relation $\sim$ on $\mathbb{R}^2$ by $(x, y) \sim (u, v)$ if and only if $x - y = u - v$. Describe the partition of $\mathbb{R}^2$ resulting from this relation.

3. For $t \in \mathbb{R}$, define $\ell_t \in \mathbb{R}^2$ by
$$\ell_t = \{(x, y) : x + y = t\}.$$
The set $\{\ell_t : t \in \mathbb{R}\}$ forms a partition of $\mathbb{R}^2$. (Exercise: prove it!) What is the equivalence relation?

4. Let S be the set of infinite sequences $(a_n)_{n \in \mathbb{N}}$ of real numbers; for instance,
$$1, 2, 3, 4, \ldots \text{ or } 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$$
are both elements of S. Define a relation $\sim$ on S by $(a_n) \sim (b_n)$ if $a_n = b_n$ for all but finitely many $n \in \mathbb{N}$.
   (a) Give an example of (a_n), (b_n) with $(a_n) \sim (b_n)$, but $(a_n) \neq (b_n)$.
   (b) Give an example of (a_n), (b_n) with $(a_n) \nsim (b_n)$.
   (c) Is $\sim$ an equivalence relation? Prove your answer.