Worksheet §10

1. Prove the triangular number (or "handshake") formula using induction:

$$\sum_{k=1}^{n-1} k = \binom{n}{2}.$$

2. Prove that if $n \geq 2$ and $A_1, \cdots, A_n$ are sets, then

$$(A_1 \cap A_2 \cap \cdots \cap A_n)^c = A_1^c \cup A_2^c \cup \cdots \cup A_n^c.$$

   Recall that ^{c} means "complement".

3. Let P, A be $n \times n$ matrices with P an invertible matrix, and let $k \geq 1$. Prove that

$$(PAP^{-1})^k = PA^kP^{-1}.$$

   Remember, matrix multiplication is not commutative!

4. For $n \geq 1$, show that $n < 2^n$.