MATH 350 Assignment 5 Solutions

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4.2

Proof. Suppose x is an odd integer. We will show that x^3 is also odd.
By definition of odd, $x = 2k + 1$ for some $k \in \mathbb{Z}$.
Thus, $x^3 = (2k + 1)^3 = 8k^3 + 12k^2 + 6k + 1 = 2(4k^3 + 6k^2 + 3k) + 1$.
So $x^3 = 2b + 1$ where b is the integer $4k^3 + 6k^2 + 3k$.
Thus $x^3 = 2b + 1$ for an integer b.
Therefore, x^3 is odd, by definition of odd. $\square$

4.4

Proof. Suppose $x, y \in \mathbb{Z}$, and that x and y are odd. We will show xy is odd.
By definition of odd, $x = 2k + 1$ and $y = 2j + 1$ for some integers j and k.
Thus, $xy = (2k + 1)(2j + 1) = 4kj + 2k + 2j + 1 = 2(2kj + k + j) + 1$.
So, $xy = 2b + 1$ where b is the integer $2kj + j + k$.
Thus, $xy = 2b + 1$ for some integer b.
Therefore, xy is odd by definition. $\square$

4.5

Proof. Suppose $x, y \in \mathbb{Z}$, and that x is even. We will show that xy is even.
By definition of even, $x = 2k$ for some $k \in \mathbb{Z}$.
Thus, $xy = (2k)y = 2(ky)$.
So, $xy = 2b$ for the integer $b = ky$.
Thus, $xy = 2b$ for an integer b.
Therefore, xy is even, by definition of even. $\square$

4.6

Proof. Suppose $a, b, c \in \mathbb{Z}$. Also assume that $a \mid b$ and $a \mid c$.
It will be shown that $a \mid (b + c)$.
By definition of divides, $b = aj$ and $c = ak$ for some $j, k \in \mathbb{Z}$.
Observe, $b + c = aj + ak = a(j + k)$.
So, $b + c = az$ for the integer $z = j + k$.
Therefore, $a \mid (b + c)$ by definition $\square$