MATH 350 Assignment 4 Solutions

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3.2.3

a.

Here we have 6 options and repetition is allowed. That means we have 6 choices for each spot in the list. Thus we have

$$6 \cdot 6 \cdot 6 = 216$$

possible lists.

b.

Now that repetition is **not** allowed, our choices for spots on the list depend on the previous. Thus we have one less choice for each spot. So there are

$$6 \cdot 5 \cdot 4 = 120$$

possible lists.

c.

Since we have three members of our list, and the position of those members matter we have three cases here where A could be.

$$\begin{bmatrix} A & 5 & 4 \end{bmatrix} - - \begin{bmatrix} 5 & A & 4 \end{bmatrix} - - \begin{bmatrix} 5 & 4 & A \end{bmatrix}$$

These are the three cases, A could be in the left, middle, or right, and since repetition is not allowed this leaves 5 choices for the next spot and then 4 for the last. Thus

$$1 \cdot 5 \cdot 4 + 5 \cdot 1 \cdot 4 + 5 \cdot 4 \cdot 1 = 60$$

is the number of possible lists.

d.

For this one it makes more sense to find how many lists have repetition and **not** have an A in them. We already calculated that there are 216 possible lists with repetition. If we removed A as a possibility, and allow repetition, we have

$$5 \cdot 5 \cdot 5 = 125 \text{ lists that do not include } A.$$

Now subtracting that from the number of total possible list yields

$$216 - 126 = 91$$

lists.

3.2.7

a.

We have 26 options for each spot, so we get

$$26^4 = 456976$$

possible lists.

b.

Since we can't have any consecutive letters be the same, after the first letter is chosen, we have one less choice for the next letter, and the pattern continues.

$$26 \cdot 25 \cdot 25 \cdot 25 = 406250$$

3.3.2

Here we have 4 suites and 13 cards in each suit, and of course there are no repetitions. For each suite we see that there are

$$13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 = 154440$$

ways to draw 5 of the same suite. Since we have four distinct suites we have

$$154440 \cdot 4 = 617760$$

total possibilities.

3.3.6

a.

For this one is it easier to find the total amount or lists and then how many of those have **no** repeating letters. The total number of lists is

$$5^5 = 3125.$$

To find how many do not have repeating letters we use

$$5! = 120.$$

Thus their difference is the number of lists that do not have repeating letters

$$3125 - 120 = 3005.$$

b.

With 5 letters and 6 spots for those letters, it is impossible[1] to create a list that doesn't repeat, so we only need to find out how many total lists there are.

$$5^6 = 15625.$$

3.3.7

Because we need *at least* one upper case letter, there can be 1,2,3,4 or 5 uppercase letters. Instead of calculating all these, we will find out how many have no uppercase letters and subtract that from the total. Note that we now have 52 options to choose from:

$$52^5 - (26^5) = 368322656$$

Now to find out how many passwords contain a mix, we again take the total and subract the passwords that are either all uppercase or lowercase:

$$52^5 - 26^5 - 26^5 = 356441280$$

3.4.7

First we figure out how many ways we can permute the 5 odd numbers and 4 even numbers without repetition.

$$\text{Odd: } 5! = 120 \text{ Even: } 4! = 24.$$

Now we multiply these together to yield

$$120 \cdot 24 = 2880.$$

[1] This fact is generalized by something called the *Pigeonhole Principle*.

3.4.8

There are 4! ways to permute the odds and 3! ways to permute the evens. Notice that there are 4 ways you can position the 4 odd numbers in our sequence. Thus we get

$$4(4! \cdot 3!) = 576$$

possible numbers.