MATH 350 Assignment 3 Solutions

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2.5.9

P	Q	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$	$\sim(\sim P \vee \sim Q)$
T	T	F	F	F	T
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	F

2.5.10

In order to do this problem without the use of a massive truth table consider what each side of the conditional statement must be to make it false.

$$T \Rightarrow U \text{ is false when } T \text{ is true and } U \text{ if false.}$$

Thus we see that two conditions must be satisfied:

$$(P \wedge Q) \vee R \text{ must be true, and}$$
$$R \vee S \text{ must be false.}$$

For $R \vee S$ to be false both R and S must be false. Since we know R to be false, and $(P \wedge Q) \vee R$ is true, we conclude that $(P \wedge Q)$ is true. Therefore, both P and Q must be true.

2.6.10

We will see if these statements are equivalent by using logic.

$$(P \Rightarrow Q) \vee R = (\sim P \vee Q) \vee R.$$

as $(P \Rightarrow Q)$ and $(\neg P \vee Q)$ are equivalent.
Now let us see the left-hand side.

$$\begin{aligned} \sim((P \wedge \sim Q) \wedge \sim R) &= \sim(P \wedge \sim Q) \vee R) && \text{(DeMorgan's Law)} \\ &= (\sim P \vee Q) \vee R. && \text{(DeMorgan's Law)} \end{aligned}$$

Observe that both statements are equivalent.

2.7.2

Here are two common ways mathematicians would interpret this statement.

$$\forall x \in \mathbb{R}, \exists n \in \mathbb{N}, x^n \geq 0.$$

"For all x in $\mathbb{R}$, there exists an n in $\mathbb{N}$ such that x^n is greater than or equal to 0."

"For every real number x, there is a natural number n where x^n is greater than or equal to 0."

This statement is true. If $x \geq 0$ we can choose any n. If $x < 0$, we choose any even $n \in \mathbb{N}$.

2.7.8

There is no one way to translate this into English, but here are two common ways mathematicians would interpret this statement.

$$\forall n \in \mathbb{Z}, \exists X \subseteq \mathbb{N}, |X| = n.$$

"For all n in $\mathbb{Z}$, there is a subset X of $\mathbb{N}$ such that $|X| = n$."

"For every integer n, there exists a subset X of the natural numbers where the cardinality of X is n."

This statement is false. Consider $n < 0$.

2.9.5

English:

For every positive number, ϵ there is a positive number δ for which $|x-a|<\delta$ implies $|f(x)-f(a)|<\epsilon$.

Symbolic:

$$\forall \epsilon \in \mathbb{R}, \epsilon > 0, \exists \delta \in \mathbb{R}, \delta > 0, \text{ such that } |x-a|<\delta \implies |f(x)-f(a)|<\epsilon.$$
$$\text{or}$$
$$\forall \epsilon \in \mathbb{R}^{>0}, \exists \delta \in \mathbb{R}^{>0}, \text{ such that } |x-a|<\delta \implies |f(x)-f(a)|<\epsilon.$$

2.10.4

One can translate this to symbolic notation first in order to negate.

$$\begin{gathered}
\sim (\forall \epsilon \in \mathbb{R}^{>0}, \exists \delta \in \mathbb{R}^{>0}, \text{ s.t. } \forall x, a \in \mathbb{R}, |x-a|<\delta \implies |f(x)-f(a)|<\epsilon) \equiv \\
\exists \epsilon \in \mathbb{R}^{>0} \text{ s.t. }, \forall \delta \in \mathbb{R}^{>0}, \exists x, a \in \mathbb{R}, \sim (|x-a|<\delta \implies |f(x)-f(a)|<\epsilon) \equiv \\
\exists \epsilon \in \mathbb{R}^{>0}, \text{ s.t. } \forall \delta \in \mathbb{R}^{>0}, \exists x, a \in \mathbb{R}, |x-a|<\delta \wedge \sim (|f(x)-f(a)|<\epsilon) \equiv \\
\exists \epsilon \in \mathbb{R}^{>0}, \text{ s.t. } \forall \delta \in \mathbb{R}^{>0}, \exists x, a \in \mathbb{R}, |x-a|<\delta \wedge |f(x)-f(a)| \geq \epsilon.
\end{gathered}$$

In English this reads: "There exists a positive number ϵ such that for every positive δ, $|x-a|<\delta$ and $|f(x)-f(a)|\geq\epsilon$."

2.10.6

Again, let us translate this to symbolic logic notation.

$$\begin{gathered}
\sim (\exists a \in \mathbb{R}, \text{ such that } \forall x \in \mathbb{R}, a+x=x) \equiv \\
\forall a \in \mathbb{R}, \exists x \in \mathbb{R}, \text{ such that } \sim (a+x=x) \equiv \\
\forall a \in \mathbb{R}, \exists x \in \mathbb{R}, \text{ such that } a+x \neq x.
\end{gathered}$$

In English we read this as: "For every real number a there is a real number x such that the sum of x and a *does not* equal x."