MATH 350 Assignment 10 Solutions

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11.1.5

{(1,2),(2,5),(3,3),(4,2),(4,3),(5,0)}

11.1.13

$R = \{(x, y) : x \neq y, x, y \in \mathbb{R}\}$

11.1.15

$R = \{(x, y) : x \equiv y \mod 3, x, y \in \mathbb{Z}\}$

11.3.8

Reflexivity

Consider $x^2 + x^2 = 2x^2$. This is even by definition. Thus xRx is true.

Symmetry

Suppose $x^2 + y^2$ is even. Since addition is commutative, $y^2 + x^2$ is even. Thus if xRy then yRx.

Transitivity

Suppose xRy and yRz. Thus $x^2 + y^2 = 2j$ and $y^2 + z^2 = 2k$ for some integers j, k.
Then $x^2 = 2j - y^2$ and $z^2 = 2k - y^2$.
Observe

$$\begin{aligned} x^2 + z^2 &= (2j - y^2) + (2k - y^2) \\ &= 2j + 2k - 2y^2 \\ &= 2(j + k - y^2). \end{aligned}$$

This is even by definition. Thus xRz is true.

Equivalence Classes

There is two equivalence classes.
$[0] = \{2k : k \in \mathbb{Z}\}$
$[1] = \{2k + 1 : k \in \mathbb{Z}\}$

11.3.11

The previous question is a counter example. It is a relation on an infinite set $\mathbb{Z}$, yet has only two equivalence classes.

11.4.2

$$\Big\{\{a\}, \{b\}, \{c\}\}, \{\{a, b\}, c\}, \{\{a, c\}, b\}, \{\{b, c\}, a\}, \{\{a, b, c\}\Big\}.$$