Name:

Math 350: Exam 6

Make sure to show all your work as clearly as possible. This includes justifying your answers if required. Calculators are not allowed.

You may use any result from the chapters covered in the text or from lecture. You may not use the results of homework or worksheet problems.

1. (10 pts) Give an example of a relation on a set S which is reflexive and transitive, but not symmetric. Make sure to specify both S and the relation!

Solution: There are many. A simple one is the relation $\leq$ on $\mathbb{R}$.

2. (10 pts) In $\mathbb{Z}_9$, fill in the entries in the partial multiplication table below. Entries should be in simplest terms.

Solution:

$\cdot$	[3]	[4]	[5]
[3]	0	3	6
[4]	3	7	2
[5]	6	2	7

3. (30 pts) Let $A = \mathbb{Q} - \{0\}$. Define a relation $\sim$ on A by $x \sim y$ if $\exists r \in A$ such that $x = r^2 y$. Prove that $\sim$ is an equivalence relation.

Solution: Reflexivity: Let $x \in A$. Then $x = 1^2 \cdot x$ and $1 \in A$, so $x \sim x$ by definition of $\sim$.

Symmetry: Suppose $x, y \in A$ and $x \sim y$. By definition of $\sim$, $\exists r \in A$ such that $x = r^2 y$. As $r \in A$, $r \neq 0$. Observe that $y = (\frac{1}{r})^2 x$. Write $r = \frac{a}{b}$; we can do this since $r \in \mathbb{Q}$. Then $\frac{1}{r} = \frac{b}{a}$; since $r \neq 0$, $a \neq 0$, so this is fine. We conclude that $\frac{1}{r} \in A$, so $y \sim x$.

Transitivity: Suppose $x, y, z \in A$ with $x \sim y$ and $y \sim z$. Then $\exists r, s \in A$ such that $x = r^2 y$ and $y = s^2 z$. Substituting, we get $x = (rs)^2 z$. But as r, s are rational, so is rs. Since $r, s \neq 0$, $rs \neq 0$ too. Therefore $rs \in A$, so $x \sim z$.