Name:

Math 350: Exam 2

Make sure to show all your work as clearly as possible. This includes justifying your answers if required. Calculators are not allowed.

You may use any result from the chapters covered in the text or from lecture. You may not use the results of homework or worksheet problems.

1. (a) (5 pts) $\cup_{i\in\mathbb{N}}[i, i+1] =$

   Solution: These are intervals: $[1, \infty)$

   (b) (5 pts) $\cap_{i\in\mathbb{N}}[i, i+1] =$

   Solution: $\varnothing$

2. (5 pts) Express $x \in A - B$ as a compound statement.

   Solution: $x \in A$ and $x \notin B$.

3. (5 pts) Suppose P is false and $(R \Rightarrow S) \Leftrightarrow (P \wedge Q)$ is true. Find the truth values of R and S.

   Solution: R is true and S is false: $P \wedge Q$ must be false, so for the biconditional to be true, $R \Rightarrow S$ must be false. There is only one way for the latter to happen, giving our answer.

4. True or false:

   (a) (5 pts) $\forall X \in \mathscr{P}(\mathbb{N}), X \subseteq \mathbb{R}$

   Solution: True

   (b) (5 pts) $\exists n \in \mathbb{N}$ such that $\forall X \in \mathscr{P}(\mathbb{N})$, $\#X \leq n$

   Solution: False: take $X = \mathbb{N}$, so then $\#X \neq n$ for any $n \in \mathbb{N}$.

   (c) (5 pts) $\forall X \in \mathscr{P}(\mathbb{N})$, $\exists n \in \mathbb{N}$ such that $\#X = n$.

   Solution: False; same example.

5. Negate the following.

   (a) (5 pts) If f is nonconstant, then $f' \neq 0$.

   Solution: f is nonconstant and $f' = 0$.

   (b) (5 pts) Every element of A has an inverse.

   Solution: $\exists x \in A$ such that x does not have an inverse.

   (c) (5 pts) The set A is both smooth and abundant.

   Solution: Either A is not smooth or not abundant.