

Name:

### Math 350: Exam 1

Make sure to show all your work as clearly as possible. This includes justifying your answers if required. Calculators are not allowed.

You may use any result from the chapters covered in the text or from lecture. You may not use the results of homework or worksheet problems.

1. Write the following in set roster notation.

(a) (6 pts)  $\{x \in \mathbb{Z} : |x| < 3\}$

**Solution:**  $\{-2, -1, 0, 1, 2\}$

(b) (6 pts)  $\{x - 1 : x \in \mathbb{R}, x^2 = 4\}$

**Solution:** The possible  $x$  values are  $-2, 2$ , so the set is  $\{-2 - 1, 2 - 1\} = \{-3, 1\}$ .

(c) (6 pts)

$$\{2n + 1 : n \in \mathbb{Z}, -2 \leq n < 0\} \times \{1, 3, 6\}$$

**Solution:** In the first set, the possible  $n$  values are  $-2, -1$ , so the first set is  $\{2 \cdot (-2) + 1, 2 \cdot (-1) + 1\} = \{-3, -1\}$ . Thus the Cartesian product is

$$\{(-3, 1), (-3, 3), (-3, 6), (-1, 1), (-1, 3), (-1, 6)\}.$$

2. Write the following in set builder notation.

(a) (6 pts)  $\{4, 7, 10, 13, \dots\}$

**Solution:**  $\{3n + 1 : n \in \mathbb{N}\}$  works.

(b) (6 pts)  $\{(1, 1), (2, 4), (3, 9), (4, 16), \dots\}$

**Solution:**  $\{(x, x^2) : x \in \mathbb{N}\}$

3. Let  $A = \{1, 2, 3, \dots, 9\}$  and  $B = \{1, 2, 3, \dots, 100\}$ . For each of the following, compute the *cardinality* of the set.

(a) (6 pts)  $A \times A \times B$

**Solution:**  $\#A = 9$  and  $\#B = 100$ , so  $\#(A \times A \times B) = 9 \cdot 9 \cdot 100 = 8100$ .

(b) (6 pts)  $A \times \mathcal{P}(\emptyset)$

**Solution:**  $\#\mathcal{P}(\emptyset) = 2^{\#\emptyset} = 2^0 = 1$ , so  $\#(A \times \mathcal{P}(\emptyset)) = 9 \cdot 1 = 9$ .

4. (8 pts) Give an example of a set  $A$  for which all of the following is true: (1)  $A$  is infinite; (2)  $A \subseteq \mathbb{Q}$ ; (3)  $A \neq \mathbb{Q}$ ; and (4)  $A \not\subseteq \mathbb{Z}$ . (Note that  $\not\subseteq$  means not a subset.) Your answer should be a *single* set.

**Solution:** There are many answers. One is  $\{\frac{n}{2} : n \in \mathbb{Z}\}$ .