Homework 9 selected solutions
Due: December 5, 2025

8.2.27 Let $H(t)$ be the temperature of the coffee after t minutes. "Proportional to the difference between its temperature and that of its surroundings" is

$$k(H-70)$$

(since the ambient temperature is 70 degrees). Thus, the first sentence becomes

$$\frac{dH}{dt} = -k(H-70).$$

The negative is because $k(H-70)$ is the rate of *cooling*, so the temperature is decreasing. (It doesn't really matter, since we could instead let $k < 0$, but this is the convention.) Additionally, we have

$$H(0) = 200 \text{ and } H(10) = 100.$$

First, we solve the differential equation. We have

$$\frac{dH}{dt} + kH = 70k.$$

The integrating factor is $e^{\int k\,dt} = e^{kt}$, so we get

$$\frac{d}{dt}\left(e^{kt}H\right) = 70ke^{kt}.$$

We integrate both sides to get

$$e^{kt}H = 70e^{kt} + C$$

so that

$$H = 70 + Ce^{-kt}.$$

As $H(0) = 200$, this implies that $200 = 70 + C$, so $C = 130$. Using $H(10) = 100$, we get

$$100 = 70 + 130e^{-10k}.$$

Solving for k yields

$$k \approx 0.1466.$$

Thus

$$H(t) \approx 70 + 130e^{-0.1466t}.$$

We want to solve $H(t) = 120$, or

$$120 = 70 + 130e^{-0.1466t}.$$

Doing so, we get $t \approx 6.52$ minutes.

8.3.1 The integrating factor is $e^{\int 1\,dx} = e^x$. We get

$$\frac{d}{dx}\left(e^x y\right) = e^{2x}.$$

Integrating, we get

$$e^x y = \frac{1}{2}e^{2x} + C$$

so

$$y = \frac{1}{2}e^x + Ce^{-x}.$$

8.5.1 Rewriting with differential operators, we get

$$(D^2 + D - 2)(y) = 0.$$

The differential operator factors as

$$(D+2)(D-1)(y) = (D-1)(D+2)(y) = 0.$$

Thus we may solve $(D-1)(y) = 0$ and $(D+2)(y) = 0$. These have solutions respectively $y = Ae^x$ and $y = Be^{-2x}$. By our theorem from class (that e^{ax} and e^{bx} are linearly independent when $a \neq b$), the solutions are linearly independent. Thus by another theorem from class (that for an order linear homogeneous ODE, the solution set is combinations of two linearly independent solutions), the full solution space is

$$y = Ae^x + Be^{-2x}.$$