Homework 6 selected solutions
Due: Friday, October 24

7.2.3 Rewriting this as
$$s = \frac{1}{2}\cos\left(\pi\left(t - \frac{8}{\pi}\right)\right),$$
we get that the amplitude is $\frac{1}{2}$, the period is $2\pi/\pi = 2$, the frequency is $\frac{1}{2}$, and the phase is $\frac{8}{\pi}$.

You did not have to include an explanation, but here it is in case you are confused: the graph is vertically stretched from $\cos x$ by a factor of $\frac{1}{2}$, hence the amplitude. As $\cos x$ has a period of 2π and this function is stretched horizontally by a factor of $\frac{1}{\pi}$, we get a period of 2. The frequency is 1 over the period. Finally, after all this stretching we shift horizontally to the right by $\frac{8}{\pi}$, yielding the phase.

A. We have
$$\begin{aligned}\langle \cos x, \sin x\rangle &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos x \sin x \, dx \\ &= \frac{1}{2\pi}\int_{u_1}^{u_2} u \, du\end{aligned}$$
where $u = \sin x$. This equals
$$\begin{aligned}\frac{1}{4\pi}u^2\Big]_{u_1}^{u_2} &= \frac{1}{4\pi}\sin^2 x\Big]_{-\pi}^{\pi} \\ &= \frac{1}{4\pi}(0-0) \\ &= 0.\end{aligned}$$

The claim follows.

Note that we could have used the double angle formula
$$\cos x \sin x = \frac{1}{2}\sin 2x$$
instead.

B. Using the fact that $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, we get

$$\begin{aligned}
\|\cos x\|^2 &= \langle \cos x, \cos x \rangle \\
&= \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos^2 x \, dx \\
&= \frac{1}{4\pi} \int_{-\pi}^{\pi} 1 + \cos 2x \, dx \\
&= \frac{1}{4\pi} \left[x + \frac{1}{2} \sin 2x \right]_{-\pi}^{\pi} \\
&= \frac{2\pi}{4\pi} \\
&= \frac{1}{2}.
\end{aligned}$$