1. Let \( z = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i \).

(a) (5 points) Compute \((z + \overline{z})/2\).

**Solution:** This is just the formula for \( \text{Re}(z) \), which in this case is \( \frac{1}{\sqrt{2}} \).

(b) (10 points) Write \( z \) in exponential form.

**Solution:** Graphing \( z \), we see that \( r = 1 \) and \( \theta = -\pi/4 \). Therefore \( z = e^{-\pi i/4} \).

(c) (10 points) Compute \( z^{60} \) in rectangular form.

**Solution:** We have
\[
\begin{align*}
z^{60} &= e^{60 \cdot (-\pi i/4)} \\
&= e^{-15\pi i} \\
&= e^{-8(2\pi i) + \pi i} \\
&= e^{\pi i} \\
&= -1.
\end{align*}
\]

2. Compute the following. Unless specified, you may use the form of your choice.

(a) (10 points) \((1 - 2i)^2\)
Solution:

\[(1 - 2i)(1 - 2i) = 1 + 4i^2 - 2i - 2i
\]
\[= -3 - 4i.\]

(b) (10 points) \(1 + \sqrt{3}i \over e^{2\pi i/5}\)

Solution: We have \(1 + \sqrt{3}i = 2e^{\pi i/3}\). Dividing, we get \(2e^{\pi i/3} - 2\pi i/5 = 2e^{-\pi i/15}\).

(c) (10 points) \(e^{\pi i/2} - e^{\pi i}\)

Solution: Note that \(e^{\pi i/2} = i\) by (for example) graphing it, and similarly \(e^{\pi i} = -1\). Therefore the answer is 1 + i.

(d) (10 points) \(e^{-2\pi i} + e^{-4\pi i} + e^{-6\pi i} + \cdots + e^{-50\pi i}\)

Solution: If \(n\) is a whole number, then \(e^{2n\pi i} = (e^{2\pi i})^n = 1\), so the above is just 1 + 1 + \cdots + 1 = 25.

3. Find all solutions for the following.

(a) (15 points) \(z^4 + 4 = 0\)

Solution: We have \(z^4 = -4\). We know that there are 4 roots. Letting \(z = re^{i\theta}\), and observing that \(-4 = 4e^{i\pi} = 4e^{i\cdot3\pi} = 4e^{i5\pi} = 4e^{i7\pi}\), we see that \(r^4 = 4\) and \(4\theta\) is one of \(\pi, 3\pi, 5\pi, 7\pi\). Thus \(r = \sqrt{2}\) and \(\theta\) is one of \(\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\). Our solutions are therefore \(\sqrt{2}e^{i\pi/4}, \sqrt{2}e^{i3\pi/4}, \sqrt{2}e^{i5\pi/4}, \sqrt{2}e^{i7\pi/4}\).

(b) (15 points) \(z^3 = 2i - 2\)

Solution: As above, let \(z = re^{i\theta}\). Observe that

\[2i - 2 = 2\sqrt{2}e^{i3\pi/4} = 2\sqrt{2}e^{i11\pi/4} = 2\sqrt{2}e^{i19\pi/4}.\]

Thus \(r^3 = 2\sqrt{2} = 2^{3/2}\), and so \(r = \sqrt{2}\). Additionally, \(3\theta = 3\pi/4\) or \(11\pi/4\) or \(19\pi/4\), and so \(\theta = \pi/4\) or \(11\pi/12\) or \(19\pi/12\). Therefore our solutions are

\(\sqrt{2}e^{i\pi/4}, \sqrt{2}e^{i11\pi/12}, \sqrt{2}e^{i19\pi/12}\).

4. (15 points) Come up with a formula for \(\text{Im}(z)\) in terms of \(z\) and \(\overline{z}\), and prove that it is true.
Solution: We have \( \text{Im}(z) = \frac{1}{2i}(z - \overline{z}) \). If \( z = x + iy \), then \( \overline{z} = x - iy \), and we get

\[
\frac{1}{2i}(z - \overline{z}) = \frac{1}{2i}(x + iy - x - iy) \\
= \frac{1}{2i}(2iy) \\
= y \\
= \text{Im}(z).
\]

This proves the identity.

5. Graph and describe with words the sets described by the following equations. Explain your answers.

(a) (10 points) \( \frac{z - \overline{z}}{2i} = 3 \).

\textbf{Solution:} This says \( \text{Im}(z) = 3 \), or in other words the line \( y = 3 \).

(b) (10 points) \( |z + 2 - i| = 4 \).

\textbf{Solution:} This is the set of points which is 4 units from \( -2 + i \), and therefore is a circle with radius 4 centered at \( -2 + i \).