1. Let $z = 1 + i$ and $w = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$.

   (a) (5 points) Compute $\frac{w + \pi}{2}$.

   **Solution:** This can be done directly, or by observing that $\frac{w + \pi}{2} = \text{Re}(w) = \frac{1}{2}$.

   (b) (5 points) Write $z$ in exponential form.

   **Solution:** Graphing $z$, we see that $r = \sqrt{2}$ and $\theta = \frac{\pi}{4}$. Thus we have $z = \sqrt{2}e^{i\pi/4}$.

   (c) (10 points) Compute $w^{50}$.

   **Solution:** Converting $w$ to exponential form, we have $r = 1$ and $\theta = \frac{2\pi}{3}$, so $w = e^{2\pi i/3}$. Then $w^{50} = e^{100\pi i/3} = e^{\frac{4\pi i}{3}}$ (by subtracting off enough multiples of $2\pi$).
(d) (10 points) Compute \(w/z\) in the form of your choice.

Solution: Exponential is easiest; from the previous two parts, \(w = e^{2\pi i/3}\) and \(z = \sqrt{2}e^{i\pi/4}\), so

\[
\frac{w}{z} = \frac{e^{2\pi i/3}}{\sqrt{2}e^{i\pi/4}} = \frac{1}{\sqrt{2}}e^{2\pi i/3 - i\pi/4} = \frac{1}{\sqrt{2}}e^{5\pi i/12}.
\]

2. Evaluate the following. Write your answer in the form specified.

(a) (10 points) \(e^{-2\pi i} - 3e^{-4\pi i} - 2e^{-6\pi i}\), rectangular

Solution: We have \(e^{-2\pi i} = e^{-4\pi i} = e^{-6\pi i} = 1\), so the answer is \(1 - 3 - 2 = -4\).

(b) (10 points) \(-e^{3\pi i/2} + 2e^{5\pi i/6}\), rectangular

Solution: Converting each to rectangular (I omit the graphs), we get

\[e^{3\pi i/2} = -i\] and \(2e^{5\pi i/6} = -\sqrt{3} + i\).

Thus the expression above is

\[-(-i) + (-\sqrt{3} + i) = -\sqrt{3} + 2i.\]

(c) (10 points) \(\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) \cdot (-i)\), exponential

Solution: We can either multiply first and then convert to exponential, or convert first and then multiply. Let’s multiply first:

\[
\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) \cdot (-i) = -\frac{i}{\sqrt{2}} - \frac{i^2}{\sqrt{2}} = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i.
\]
3. Solve each of the following for $z$ over the complex numbers.

(a) (10 points) $z^2 - z + 1 = 0$

**Solution:** Applying the quadratic formula, we obtain

$$z = \frac{1 \pm \sqrt{(-1)^2 - 4 \cdot 1}}{2} = \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm \sqrt{3}i}{2}.$$

(b) (10 points) $z^3 = i$

**Solution:** Let $z = re^{i\theta}$. Observe that $i = e^{i\pi/2} = e^{i5\pi/2} = e^{i9\pi/2}$. Substituting, we obtain

$$r^3 e^{i3\theta} = \begin{cases} e^{i\pi/2} & \text{or} \\ e^{i5\pi/2} & \text{or} \\ e^{i9\pi/2}. \end{cases}$$

In every case, $r^3 = 1$ so $r = 1$. As for $\theta$, we get $3\theta = \pi/2$ or $5\pi/2$ or $9\pi/2$, so $\theta = \pi/6$ or $5\pi/6$ or $3\pi/2$. Thus our answers are

$$z = e^{\pi i/6} \text{ or } e^{5\pi i/6} \text{ or } e^{3\pi i/2}.$$

4. (15 points) Let $z = x + iy$. Show that $\Im(z) = \frac{z - \bar{z}}{2i}$.

**Solution:** We have $\bar{z} = x - iy$, so

$$\frac{z - \bar{z}}{2i} = \frac{(x + iy) - (x - iy)}{2i} = \frac{x + iy - x + iy}{2i} = \frac{2iy}{2i} = y = \Im(z).$$
5. Graph the following. Make sure to justify your answers.

(a) (10 points) \(|z + i| = 2\)

\[\textbf{Solution:} \text{ Rewriting } |z+i| = |z-(−i)|, \text{ this is the set of points 2 units from } −i, \text{ which is a circle of radius 2 centered at } −i.\]

(b) (10 points) \(z − \overline{z} = 2i\)

\[\textbf{Solution:} \text{ Rewriting as } \frac{z - \overline{z}}{2i} = 1, \text{ by problem 4, this is the same as } \text{Im}(z) = 1, \text{ which is a horizontal line through } i.\]