Name:

Math 330: Exam 1

September 27, 2023

Make sure to show all your work as clearly as possible. This includes justifying your answers if required. Avoid using the back of the page; instead, there is an extra sheet at the end that you can use. Calculators are not allowed.

1. (5 points) Write 132 in Egyptian hieroglyphics.

Solution: $||\ \cap\cap\cap$ 𓍢

2. Compute the following using Egyptian methods.

(a) (5 points) 9×5

Solution: We construct the following table:

1	5
2	10
4	20
8	40

Notice that $1 + 8 = 9$. The sum of the corresponding elements in the second column is $5 + 40 = 45$.

(b) (5 points) 25×3

Solution: Using the same method:

1	3
2	6
4	12
8	24
16	48

Notice that $16 + 8 + 1 = 25$, so taking the corresponding values in the second column, we get $48 + 24 + 3 = 75$.

(c) (5 points) $91 \div 7$

Solution: You guessed it: two columns!

1	7
2	14
4	28
8	56

We have $91 = 56 + 28 + 1$, so the quotient is the sum of the corresponding values in the the first column; that is, $8 + 4 + 1 = 13$.

(d) (5 points) $6 \times \overline{12}$

Solution: Two columns all the way:

$$\begin{array}{cc} 1 & \overline{12} \\ 2 & \overline{6} \\ 4 & \overline{3} \end{array}$$

We have $4 + 2 = 6$, so the product is $\overline{3}\,\overline{6}$. Note that this equals $\overline{2}$, but you didn't have to simplify.

3. (10 points) Convert $\frac{11}{12}$ into an Egyptian fraction.

Solution: We apply the greedy algorithm to get $\frac{11}{12} = \frac{2}{3} + \frac{1}{4} = \overline{\overline{3}}\,\overline{4}$.

4. (5 points) What is the name of the papyrus that most of our knowledge of ancient Egyptian mathematics comes from?

Solution: The Rhind papyrus

4. ______________

5. (10 points) Convert 172 into Babylonian (that is, sexagesimal).

Solution: $172 = 2 \cdot 60 + 52 = 2, 52$

6. (10 points) Convert $\frac{5}{12}$ into sexagesimal.

Solution: We have

$$\begin{aligned} \frac{5}{12} &= \frac{25}{60} \\ &=; 25 \end{aligned}$$

7. (5 points) Give an example of a 3 digit number which is regular (in the Babylonian sense).

7. ______________

Solution: There are a lot of answers; 100 for example works because its only prime factors are 2 and 5.

8. (5 points) Give an example of a 3 digit number which is *not* regular.

8. _______________

Solution: Also a lot of answers; 700 works because 7 is a prime factor.

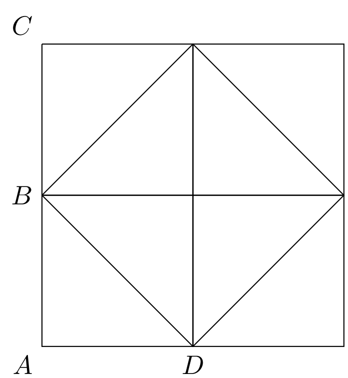
9. (10 points) Give an estimate of $\sqrt{103}$ using one step of the Babylonian method. Give your answer in regular Arabic numbers; that is, do not use Babylonian numbers (unless you are dying to do so!).

Solution: We guess $a = 10$. Then $b = 103 - 10^2 = 3$. Our new guess is

$$c = a + \frac{1}{2}\frac{1}{a}b = 10 + \frac{1}{20}3 = 10.15.$$

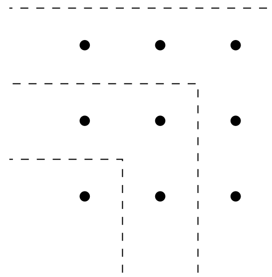
10. (10 points) Draw the picture the Pythagoreans used to prove that $\sqrt{2}$ is irrational. Make sure to mark relevant side lengths and identify special shapes (for instance, if a triangle is equilateral, state that clearly).

Solution: Everything that looks like a square in the diagram is a square. In the proof, the Pythagoreans start with one set of lengths, and then scale it to get another set of lengths; either version is fine. If you did the first version, then $AB = 1$ and $BD = \sqrt{2}$ (all other lengths are deducible from these). If you did the second version, then $AB = n$ and $BD = m$, where we assume for the sake of contradiction that $\sqrt{2} = \frac{m}{n}$ in lowest terms.



11. (5 points) Draw the picture the Pythagoreans used to demonstrate that $1 + 3 + 5 = 9$.

Solution: In the picture, there are a total of 9 dots, but the shells respectively contain $1, 3$, and 5 dots.



12. Below, describe any 3 of the 5 Platonic solids by name and number of faces.

> **Solution:** These are the tetrahedron with 4 faces, the cube with 6, the octahedron with 8, the dodecahedron with 12, and the icosahedron with 20.

13. (5 points) The book for which Euclid is most famous contains an axiomatic treatment of geometry, as well as number theory and other topics. What is its name?

13. ________________

> **Solution:** The Elements

14. (5 points) In what city is Euclid believed to have lived?

14. ________________

> **Solution:** Alexandria

15. The following questions concern the proof of the Pythagorean Theorem for the right triangle $\triangle ABC$. Two identical figures are provided. For (a) and (b), draw on Figure 1. For (c) and (d), use Figure 2.

    (a) (5 points) In the part of the proof shown in class, it was shown that the area of the square $ABFG$ equals the area of part of square $BCED$. Draw in the relevant part of square $BCED$, and explain precisely how it was drawn.

    > **Solution:** Draw a line parallel to $\overline{BD}$ through A. This divides square $BCED$ into two rectangles. The relevant part is the top rectangle.

    (b) (5 points) On the same figure, draw and identify the two triangles which you use for the part of the proof described in part (a) above.

    > **Solution:** These are $\triangle ABD$ and $\triangle FBC$ indicated below.

    (c) (5 points) To complete the proof of the Pythagorean Theorem, one would show that square $ACKH$ has the same area as part of square $BCED$. As in (a), identify which part.

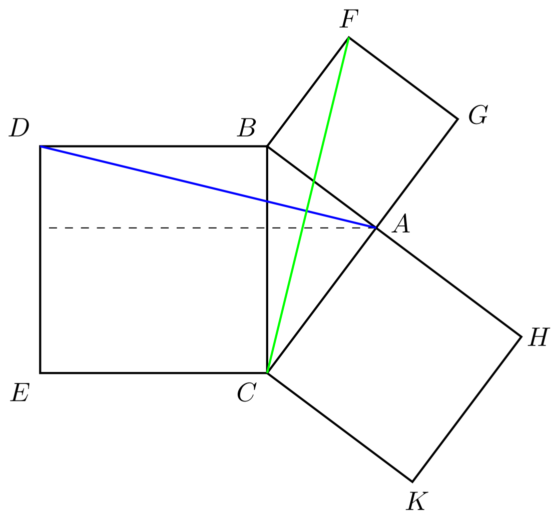


Figure 1: For parts (a) and (b)

Solution: Draw the same line as in (a), but now we take the bottom rectangle.

(d) (5 points) Draw and identify the two triangles which you would use for the part of the proof described in (c).

Solution: They are $\triangle ACE$ and $\triangle KCB$.

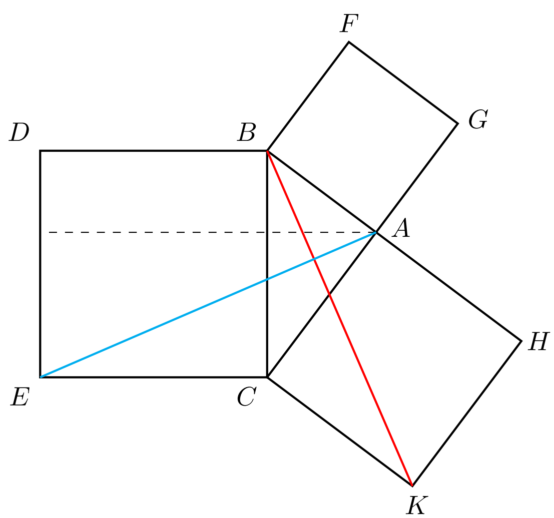


Figure 2: For parts (c) and (d)