Exam 3 study guide

Exam 3 will be on Thursday, April 24. This exam will be covering Sections 3.6 and 4.1–4.4. The only exceptions are that we did not cover Theorems 4.11, 4.12, and similarity (p. 301 and 302). Of course you still need to know/be able to use the ideas from Exam 1 (Chapter 1 and 2), even though they may not be tested explicitly. For example, even though I will not ask you to define the dot product on Exam 2, you may still have to use the dot product and its properties to solve some problems.

You will be allowed to use MATLAB on the classroom computer. Whenever you compute something by MATLAB, like rref(A), then indicate this by writing ML right next to it. If it is a simple operation, like multiplying two matrices, or adding two vectors (which one could do just as easily by hand), then this is not necessary, but anything more involved (like finding an inverse matrix) requires it for full credit! You may use any of the material from the MATLAB Quick intro sheet. If you find any other operations on MATLAB, then please check with me before the exam that it is okay to use them.

There will be a proof type question on the exam, but there will be more computational questions than proofs. Make sure that you understand the homework problems, since they give a pretty good cross section of what you have to know.

If you have time for it, then doing extra problems from the book is the way to go. Here are some sample problems: Chapter 3 review questions (page 252): 1, 18–20; Chapter 4 review questions (page 365): 1a–h, 2–4, 6–12, 16–19

Below is an exam that was given last year.

1. Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation giveny by first reflecting across the x-axis, the rotating $30°$ clockwise about the origin, followed by reflection across the y-axis. Find the standard matrix $[T]$.

2. Suppose the 3×3 matrix A has eigenvectors

$$\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \text{ and } \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix}$$

   with respective eigenvalues $-3, 1, 0$. Let $\vec{v} = [7, 4, -10]$. Find $A^{100}\vec{v}$.

3. Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation with $T\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -8 \\ 4 \end{bmatrix}$ and $T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 17 \end{bmatrix}$.

   (a) Find $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$.

   (b) Find $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$.

(c) Find the standard matrix $[T]$.

4. Find all k such that the matrix

$$A = \begin{bmatrix} 2 & k & 0 \\ 5-k & 3 & 0 \\ 2 & 5 & k^2 \end{bmatrix}$$

is not invertible. Justify your answer.

5. Suppose that

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 4.$$

Find the following determinants with justification.

(a) $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g-3a & h-3b & i-3c \end{bmatrix}$

(b) $\det \begin{bmatrix} d & e & f \\ a & b & c \\ 5g & 5h & 5i \end{bmatrix}$

6. Let

$$A = \begin{bmatrix} 3 & 0 & 3 & 0 & 3 & 3 \\ 0 & 3 & 3 & 3 & 0 & 3 \\ 0 & 0 & 4 & 4 & 4 & 4 \\ 0 & 0 & 0 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{bmatrix}.$$

(a) Find the characteristic polynomial of A.

(b) Find the eigenvalues of A.

(c) For each eigenvalue, find a basis for the eigenspace.

7. True or false.

(a) If $T : \mathbb{R}^6 \to \mathbb{R}^7$ is a linear transformation, then there is a 7×6 matrix A such that $T(\vec{x}) = A\vec{x}$ for all $\vec{x} \in \mathbb{R}^6$.

(b) If A is invertible, then $\det(A^{-1}) = \det(A^T)$.

(c) If $\det A = 4$ and $\det B = 8$, then $\det(A + B) = 12$.

8. True or false, and justify your answer.

(a) The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(\vec{x}) = -4\vec{x}$ is a linear transformation.

(b) If 1 is the only eigenvalue of a lower triangular 3×3 matrix A, then A must be the identity matrix.

(c) If A and B are $n \times n$ matrices whose columns are the same except in a different order, then $\det(A) = \pm \det(B)$.

(d) If A is a 5×5 matrix, then $\det(-A) = -\det(A)$.

(e) Two eigenvectors of the same eigenvalue must be linearly independent.