Exam 2 study guide

Exam 2 will be on Thursday, March 27. This exam will be covering Sections 3.1–3.3 and 3.5. The only exceptions are that we did not cover Theorems 3.5, 3.25 and 3.28. Of course you still need to know/be able to use the ideas from Exam 1 (Chapter 1 and 2), even though they may not be tested explicitly. For example, even though I will not ask you to define the dot product on Exam 2, you may still have to use the dot product and its properties to solve some problem.

You will be allowed to use MATLAB on the classroom computer. Whenever you compute something by MATLAB, like rref(A), then indicate this by writing ML right next to it. If it is a simple operation, like multiplying two matrices, or adding two vectors (which one could do just as easily by hand), then this is not necessary, but anything more involved (like finding an inverse matrix) requires it for full credit! You may use any of the material from the MATLAB Quick intro sheet. If you find any other operations on MATLAB, then please check with me before the exam that it is okay to use them.

There will be a proof type question on the exam, but there will be more computational questions than proofs. Make sure that you understand the homework problems, since they give a pretty good cross section of what you have to know.

If you have the time for it, then doing extra problems from the book is the way to go. Here are some sample problems, at least one of which will be on the exam: Chapter 3 review questions (page 252): 1–6, 8–11, 13, 15, 16.

Below is an exam that was given last year.

1. Verify that if $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a set of three vectors in $\mathbb{R}^7$, then $\text{Span}(S)$ satisfies the definition of a subspace of $\mathbb{R}^7$.

2. Show that the set S of all vectors $\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$ such that $x + y$ is an integer is not a subspace of $\mathbb{R}^2$.

3. Find the reduced row echelon form of the matrix A below. Use it to find the rank and nullity of A, as well as bases for the column space and null space of A. Show work and explain your reasoning. (Hint for data entry: This breaks down into simple 3×3 blocks.)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

4. Find a basis for $row(A)$ for the matrix in the previous problem. Explain how you got your answer, but you do not have to copy down any work you did in MATLAB.

5. Let A, B be $n \times n$ matrices, where B is invertible. Solve for A: $B^{-2}AB - I_n = O_{n,n}$.

6. Solve $A\vec{x} = \vec{b}$ when

$$A = \begin{bmatrix} 1 & 3 & 5 & 7 & 1 \\ 0 & 2 & 4 & 6 & -1 \\ 1 & 3 & -1 & 3 & 2 \\ 3 & 4 & 5 & 7 & 3 \\ 4 & 5 & 6 & 7 & 5 \end{bmatrix}$$

for all of the following choices of $\vec{b}$:

$$\vec{b} = \begin{bmatrix} 1 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 1 \\ s \\ 2 \\ t \\ 4 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} \sqrt{1} \\ \sqrt{2} \\ \sqrt{4} \\ \sqrt{8} \\ \sqrt{16} \end{bmatrix}$$

7. True or false?

   (a) The inverse of an elementary matrix is an elementary matrix.

   (b) If A is a square matrix, then $A^T A$ is invertible if and only if A is invertible.

   (c) If A is an $m \times n$ matrix, then the nullspace of A is a subspace of $\mathbb{R}^m$.

8. True or false? Justify your answer.

   (a) There is a 3×2 matrix with nullity 0.

   (b) There is a 3×4 matrix A such that $AB = BA$ for every 4×3 matrix B.

   (c) There is a 2×3 matrix with rank 3.

   (d) If the last row of an $n \times n$ matrix A is the sum of the other rows, then A is not invertible.

   (e) If $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \end{bmatrix}$, then there is a matrix X with $XA = B$.