Exam 1 study guide

The exam covers to §1.1–1.3 and §2.1–2.3 in the book, except that we did not cover anything related to modular arithmetic, and we skipped Theorem 1.4 (Cauchy-Schwarz Inequality). Any results that we didn't cover in class yet, that are from Chapter 3 and beyond or that are not in these sections of the textbook (such as more general results about systems of equations or vectors) can not be used on the exam. You will not be allowed to use MATLAB or a calculator on this exam.

You must know all the basic properties of scalar multiplication, vector addition/subtraction, dot product, orthogonality, lines and planes in $\mathbb{R}^2$ and $\mathbb{R}^3$, (reduced) row echelon forms and solving systems of linear equations. Spanning and linear independence will be on the exam, but not heavily.

You should be familiar with all the terminology we are using. You should know the PROPER definitions of all the concepts and objects we have been working with, and I may test these by asking for them outright. You should know the correct statements of the important results we have proved so far, so that you can apply them correctly. You do not need to memorize the statement/definition numbers. The emphasis on the exam will be on understanding of the subject material. There will be some computational questions, but there will also be at least one (short) proof.

Make sure that you understand the homework problems, since they give a pretty good cross section of what you have to know. The best way to study is to do extra problems from the book.

The exam will have some True-False questions, like the ones on page 55, number 1 and page 134 numbers 1 and 14. Below is last semester's Exam 1. I will answer questions about it in the review session.

1. Solve the following system of equations using Gaussian elimination:

$$\begin{aligned} x + y + z &= 0 \\ 3x + y + 2z &= 1 \\ 2x - 2y \quad\;\; &= 3. \end{aligned}$$

2. Solve the following system using Gaussian elimination.

$$\begin{aligned} 3x + y + 2z &= 0 \\ x + y + z &= 0 \\ 2x \quad\;\; + z &= 0 \end{aligned}$$

3. In $\mathbb{R}^3$, let $A = (3, 0, 1)$, $B = (5, 1, -1)$, and $C = (4, 3, 1)$.

   (a) Compute the distance from B to C.

   (b) Compute $\vec{AB}$, $\vec{BC}$, and $\vec{AC}$.

(c) Compute $\vec{AB} \cdot \vec{AC}$.

(d) Determine if the angle between $\vec{AB}$ and $\vec{AC}$ is obtuse, acute, or right. Justify your answer.

(e) Find a parametric equation for the plane through A, B, and C.

4. Find the distance between $P = (1, 3, 4)$ and the plane given by $2x - 3y + z = 5$.

5. Let θ be the angle formed by the line through $(1, 1, 1)$ and $(1, 4, 5)$, and the line through $(3, 2, 3)$ and $(5, 3, 5)$. Find $\cos\theta$.

6. Prove that for all vectors $\vec{v}, \vec{w} \in \mathbb{R}^n$ and $c \in \mathbb{R}$, we have $(c\vec{v}) \cdot \vec{w} = c(\vec{v} \cdot \vec{w})$.

7. Give the following definitions. Your answers should be precise and algebraic.

   (a) For $\vec{v}, \vec{w}$ in $\mathbb{R}^n$, give the projection formula for the projection of $\vec{v}$ onto $\vec{w}$, otherwise known as $\text{proj}_{\vec{w}}\, \vec{v}$.

   (b) Given vectors $\vec{v}_1, \ldots, \vec{v}_k$, give the *span* of these vectors.

8. True or false; no explanation is needed.

   (a) Every system of linear equations has zero, one, or infinitely many solutions.

   (b) A matrix in row echelon form must have a row of zeros at the bottom.

   (c) Multiplying a row of an augmented matrix by any constant does not change the solution set of the corresponding systems of equations.

   (d) If $\vec{v}$ is in $\mathbb{R}^n$ and c is a scalar, then $\|c\vec{v}\| = c\|\vec{v}\|$.

   (e) Given vectors $\vec{u}, \vec{v}, \vec{w}$ in $\mathbb{R}^n$, if $\vec{u} \cdot \vec{w} = \vec{v} \cdot \vec{w}$, then $\vec{u} = \vec{v}$.

9. Find scalars s, t such that

$$s \begin{bmatrix} 3 \\ -2 \end{bmatrix} + t \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$