# Assignment 6

due: On gradescope 11:00PM, Thursday, April 24, 2025

1. Use the method of Example 4.5 to find the eigenvalues and eigenspaces of $A = \begin{bmatrix} 7 & 9 \\ 2 & 0 \end{bmatrix}$ and then explain what this transformation does graphically like in Figure 4.8.

2. Use the method of Example 4.5 to find the eigenvalues and eigenspaces of $A = \begin{bmatrix} 3 & 7 \\ 0 & 3 \end{bmatrix}$ and then explain what this transformation does graphically like in Figure 4.8.

3. The *trace* of a matrix $A$, denoted by $\text{tr}(A)$, is the sum of the entries on its diagonal. In other words, $\text{tr}([a_{ij}]_{n\times n}) = a_{11} + a_{22} + \cdots + a_{nn}$.

   Show that the eigenvalues $\lambda$ of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ are the solutions of the quadratic equation

   $$\lambda^2 - \text{tr}(A)\lambda + \det(A) = 0.$$

4. Show that if $\lambda_1$ and $\lambda_2$ are the eigenvalues of the matrix $A$ in Problem 3, then $\text{tr}(A) = \lambda_1 + \lambda_2$ and $\det(A) = \lambda_1\lambda_2$.

5. Use cofactor expansion along any row or columns to find the following determinant. Show work!

   $$\begin{vmatrix} a & 0 & b & 0 \\ 0 & c & 0 & d \\ e & 0 & f & 0 \\ 0 & g & 0 & i \end{vmatrix}$$

6. Consider a matrix $A$ such that $A^3 = A$. Find all possible values of the determinant of $A$.

7. For $A = \begin{bmatrix} 1 & -2 \\ 7 & 10 \end{bmatrix}$ find the characteristic polynomial, eigenvalues (with algebraic and geometric multiplicity) and a basis for each of the eigenspaces.

8. Let $A = \begin{bmatrix} -1/3 & 2/3 & 2/3 \\ 0 & 1/3 & 2/3 \\ 0 & 0 & 1 \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$. Find $A^{20}\vec{x}$ using Theorem 4.19.

9. A diagonalization of a matrix $A$ is given in the format $D = P^{-1}AP$ below. Find the eigenvalues of $A$ and list a basis for each of the corresponding eigenspaces.

   $$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & 0 & -1 \end{bmatrix}$$

10. Let $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$. Show and explain why $A$ is not diagonalizable.

11. State your outside sources and who you worked with on this assignment. (If you didn't work with anybody and didn't use any other outside sources, clearly write NO OUTSIDE SOURCES.)