Given a fixed set $S \subseteq U$ with $\#U = p$ and $\#S = n$, we let

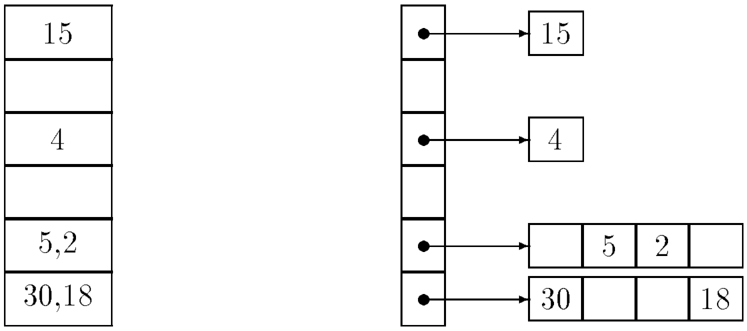
$$\mathcal{H}_r = \{(ax \bmod p) \bmod r \mid a \in \{1, \ldots, p-1\}\}.$$

We pick the first-level hash function $h \in \mathcal{H}_n$ at random. Then, we define the *bucket size*

$$b_k = \#\{x \in S \mid h(x) = k\} \text{ for } 0 \leq k \leq n.$$

For each bucket $k$, we choose the second-level hash function $h_k$ at random from $\mathcal{H}_{b_k^2}$. (In the assignment, it says to use $\mathcal{H}_{2b_k^2}$, but this makes the example too large.)

Let $U = [31] = \{0, 1, \ldots, 30\}$ and $S = \{2, 4, 5, 15, 18, 30\}$. Say $n = 6$ and $h(x) = (2x \bmod 31) \bmod 6$. Then, the first level hash function maps the elements of $S$ into buckets as shown in Figure 1(a). There are four hash functions at the second level. The functions $h_0$ and $h_2$ are trivial, mapping everything to 0. Formally, $h_0(x) = h_2(x) = (1 \cdot x \bmod 31) \bmod 1$. Since $b_4^2 = b_5^2 = 4$, it follows that $h_4, h_5 \in \mathcal{H}_4$. Say we choose $h_4(x) = (x \bmod 31) \bmod 4$ and $h_5(x) = (3x \bmod 31) \bmod 4$. Then, we get the pictorial representation shown in Figure 1(b) for the two-level hash table.




(a) (b)

Figure 1: A Two-Level Hash Table.

Now, say we want to actually represent the two-level hash table as a single array. One way to do this is as follows: The first two locations contain the values of $n$ and $a$ (to specify $h$). The next $n$ locations contain pointers to the portions of the array corresponding to each second-level table. Then, each (nonempty) second-level table is given, preceded by the specification of its hash function. The following figure should help make this clearer by giving the representation for the example above.

| | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 2 | 9 | | 12 | | 15 | 21 | 1 | 1 | 15 | 1 | 1 | 4 | 4 | 1 | | 5 | 2 | | 4 | 3 | 30 | | | 18 |