CSC 378F 2000
Enriched Assignment 1

due: Wednesday, September 27, 2000

1. Consider the abstract data type CACHE. An object of this abstract data type is a subset C of $\{1, ..., m\}$ of size at most k. Initially, $C = \emptyset$. The only operation is *access*, which takes a parameter i. It adds i to C and, if this causes the size of C to become bigger than k, removes the element from C that was least recently the parameter of an access operation.

   Give a data structure for this abstract data type that uses $O(k \log m)$ space (measured in bits) and has worst case time complexity $O(\log k)$. Prove that your data structure is correct and satisfies the desired complexity bounds.

2. CLR problem 8-5 (pages 170-171).

3. Consider the following randomized algorithm which takes as input an integer n and an array x containing n integers, all distinct.

   randomly choose $i \in \{1, \ldots, n\}$
   $y \leftarrow x[i]$
   $j \leftarrow 1$
   while $x[j] < y$ do $j \rightarrow j + 1$

   (a) Give an expression for the expected number of comparisons performed by this algorithm, assuming that the elements in x are equally likely to be in any order. Justify your answer.

   (b) What is the worst case expected number of comparisons performed by this algorithm? Justify your answer.

4. CLR exercise 13.4-1 (page 259).