Option Contracts and Capacity Management—Enabling Price Discrimination under Demand Uncertainty

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We explore using an option contract as a price discrimination tool under demand uncertainty. In our capacity game model, a monopolistic supplier has to build capacity before observing the uncertain demand. The demand is generated by two potential customers, who privately know their own types. The types could be either high or low, differing in willingness to pay for each unit of demand. To discriminate between the customer types, the supplier designs option contracts so that only the high type will buy options in advance. The high type will do so because the options can hedge their risk of demand loss when capacity is tight. The supplier profits in three ways. First, the high type customers pay higher marginal prices on average. Second, the high type customers’ demand is satisfied as a first priority, guaranteeing allocation efficiency. Third, the supplier can observe the number of options being purchased and so determine customer types, improving capacity investment efficiency. We compare our results to those of classical second degree price discrimination. We show that our proposed framework guarantees the same level of supplier profit even when the supplier cannot discriminate between the customers by bundling products.

Key words: option contracts; capacity management; price discrimination; demand uncertainty; monopoly revenue management
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1. Introduction

Real options contracts are used in supply chain management to protect risk-averse partners from potential uncertainties, such as demand and material cost changes (see e.g., Huchzermeier and Cohen 1996). In an incomplete contract setup, options can also improve contracting efficiency by solving the hold-up problem (see e.g., Noldeke and Schmidt 1995). In this paper, we explore using options as a price discrimination tool. In the classic economics literature, price discrimination relies on the supplier’s ability to determine customers’ different levels of willingness to pay and to charge different prices. When customer types are not observable, the supplier can offer a menu of bundles for the customers to choose from. This practice is known as second degree price discrimination (Tirole, 1988).

When customer demand is stochastic, bundling is an inefficient discrimination tool. This is because the customers’ desired quantities vary according to their realized demand. In addition, when the capacity must be determined before the demand is fully revealed, the supplier’s optimal capacity decision is critical to revenue management. Moreover, the supplier always seeks ways to better utilize tight capacity.

In this paper, we propose using a new form of option contract to improve the supplier’s revenue. In situations when capacity may be tight, customers have an incentive to hedge the risk of demand loss. The incentive is higher for those with higher willingness to
pay. To meet customers’ hedging demands, the supplier could provide option contracts where customers have the right but not the obligation to exercise. Exercising an option guarantees the availability of one unit of demand. If the option is priced so that only customers with higher willingness to pay will buy it, then those customers will self-select to pay more than their counterparts and so ensure execution of their demand.

To demonstrate the effectiveness of option contracts, we present a game theoretic model where one monopolistic supplier or service provider (he) faces two customers each with uncertain future demand. Each customer (she) has private information about their willingness to pay for one unit of demand. The supplier has to build capacity before the customers’ demand is realized. Since capacity may be insufficient for the highest level of demand realization, customers suffer from potential demand losses. When demand exceeds capacity, the supplier can only serve the demand randomly.

Option contracts can be adopted in the following manner. The supplier opens the option market to the customers before the capacity investment and uncertain demand realization. At that time, customers can purchase options with unit price \( p_o \). After observing customers’ option purchase decisions, the supplier invests in capacity. The customers then find out their actual demand, observe the supplier’s capacity and decide whether to exercise their options. Each customer pays a strike price, \( p_s \), for each unit option exercised. The amount of demand protected by the options (referred to as the “option demand” in the context) will be satisfied as the first priority. The remaining demand will be satisfied at a unit price \( p \) if there is leftover capacity.

Our proposed framework improves the supplier’s revenue in three ways. First, customers with a higher willingness to pay will pay a higher unit price when demand is tight, increasing the supplier’s overall revenue. Second, the customers’ option demands are satisfied as a first priority. The remaining demand will be satisfied only when there is extra capacity. Third, customers’ options purchases reveal their types. This knowledge allows the supplier to more efficiently adjust capacity levels, better accommodating the potential demand. The last two effects also improve the supplier’s decision efficiency, leading to an enhanced overall social welfare.

In order to successfully induce high type customers into purchasing options, the supplier needs to convince the customers that capacity could be insufficient. If a supplier is able to change capacity after the options are purchased, however, it may undermine the customers’ initial incentives to purchase them. This is because customers know the supplier will want to guarantee enough capacity to meet the demand, so as to maximize the revenue by serving as much customer demand as possible. Based on knowing their own types and the supplier’s capacity cost, a rational customer can always conjecture the capacity level that the supplier will invest in contingent on their counterpart’s type. They can therefore calculate the overall benefit of purchasing the options based on these “rationally expected” capacity levels. The supplier cannot mislead the customers. However, the capacity investment levels could be different if the supplier adopts different option contracts (e.g., different \( p_o \) and \( p_s \)). Hence, the option contract design is critical in our framework.

The supplier can decide not to build sufficient capacity to guarantee the execution of the entire option demand. The decision depends on what commitment the supplier makes in the option contracts when he fails to meet the option demand. If the supplier does not compensate unsatisfied customers, the customers will have less incentive to purchase the options. This disappoints the high type customers and reduces their valuation of the options. If the supplier promises too high a compensation, he has to always ensure to over-invest in capacity. This may be inefficient especially when the capacity cost is high. In this paper, we suggest the supplier offer an option buy-back price as a compensation mechanism which leaves the high type customers indifferent about exercising their options or selling them back to the supplier. We will show that this buy-back price scheme reduces the high type customers’ strategic decision of exercising the options and induces the supplier’s efficient capacity investment which maximizes ex ante social welfare.

Our option framework can improve revenue management in many industries wherever demand uncertainty and information asymmetry exist. One potential application is in network traffic management. Since business communication relies heavily upon emails and video conferences, network congestion can cause severe economic losses. Companies are willing to pay a premium more than what the regular users pay to ensure important business emails being delivered promptly.

Another potential application of the option framework is in the ticket sale business. Many people want to go to a concert or a game, but face the risk of not being able to attend ex post. They do not want to pay the full price for the tickets in advance because they do not want to waste money if they are unable to attend. However, if they wait too long, the tickets could be sold out. The option contract is a good solution for these people. Similar applications can be implemented in airline ticket management, hospital facility management, and the hotel reservations business. In hotel reservation business, customers who care more about getting the hotel room can make reservations before
However, in practice, reservation is often free (i.e., option price \( p_o \) is zero). This paper provides an appropriate analytical framework to determine the option price.

In the option framework, the supplier employs price discrimination by extending the customers’ decision problem into an intertemporal one. That is, the customers have to decide whether and how many options they should purchase to hedge the future risk of demand loss before their demand realization and the supplier’s capacity investment. To make the decision, they have to figure out the capacity level the supplier may invest in and the possibility they will use the options. When they evaluate the options, rational customers also take into account the fact that their option purchase reveals their types to the supplier and affects the capacity level.

An alternative pricing scheme is the spot pricing scheme, which suggests that the price should be dynamically adjusted according to congestion levels. Gupta et al. (1996 and 1999) suggest using priority classes with different spot prices to efficiently allocate network resource. The idea is that the customers with higher delay costs can choose to send their demand to the network with lower expected delay time. Afche (2004) suggests to add strategic delay in the queue to further discrimination customers and maximize the revenue of the supplier. However, under the spot pricing mechanism, customers decide whether to execute the demand when observing the spot price. The supplier cannot discover the customers’ type distribution before adjusting the capacity.

Maskin and Riley (1984) have proposed a mechanism for price discrimination without demand uncertainty. In their mechanism, the monopolist decides the allocation rules and pricing mechanism according to each customer’s reported willingness to pay. The customers then decide what to report. In a truth-telling mechanism, every customer finds it optimal to report their true private information. Deshpande and Schwartz (2002) extend the mechanism into a constrained capacity setup. In this mechanism, non-linear pricing rules are adopted to guarantee incentive compatibility. The fact that demand is uncertain in our setup changes the favorite allocations for the two types of customers. In this sense, Maskin and Riley’s mechanism cannot be directly applied to solve our problem. Boyaci and Ray (2006) discussed the impact of capacity costs on product differentiation in a product delivery model.

The literature on demand uncertainty and constrained capacity is prodigious. As an example, Birge (2000) used option pricing theory to quantify the risks associated with capacity planning under uncertainty. Özer and Wei (2004) look at managing demand uncertainty of non-perishable product with a capacitated inventory model. Sodhi (2004) compares managing demand risks under a deterministic model to that under a stochastic model. Iyer et al. (2003) proposed a demand postponement model to handle demand surges for perishable products. They justified the value of demand postponement, in which the supplier can choose to satisfy only a fraction of the aggregate demand when new information is incorporated. However, they only examined demand surge on an aggregate level. In addition, demand will be postponed randomly under demand surge. If the customers have different postponement preferences, there is potential for designing a revelation mechanism to find customers with lower delay costs. This would save the fees paid by suppliers to compensate for the postponed demand. Our framework complements the postponement mechanism by differentiating among customers’ delay costs.

To illustrate the implementation of option contracts, we present a two-period game-theoretic model. The model has one monopolistic supplier and two potential customers with private valuation of the service. Figure 1 shows the time line of events.

In the first period—the contracting period—the monopolistic supplier announces the option price, \( p_o \), and the strike price, \( p_s \), to maximize his expected overall profit. Then each customer decides the number of options to purchase, \( O_i \), according to their types.

Before the beginning of the second period—the consumption period—the supplier observes the customers’ options purchases and decides on an optimal capacity investment, \( K \). Afterwards, the customers demand, \( D_i \), is realized. Each customer decides how many options they are going to exercise (\( D_f \)) based on the observation of the aggregate demand and the capacity level. The supplier satisfies the option demand as a first priority. If \( D^o > K \), some of the options cannot be satisfied and the supplier will compensate the customers. If \( D^o < K \), the extra capacity will be used to satisfy the remaining demand.

The rest of the paper is organized as follows. Section 2 presents the model and analyzes the equilibrium strategies of the supplier and two customers. We compare the model outcome to two benchmark cases. In one of the benchmarks, the supplier cannot distinguish the customer types at all. In the other, the supplier can determine each customer’s type and can charge different prices to each of them. In Section 3, we discuss the implications and extensions of our model. Section 4 concludes the paper.

### 2. The Model

A monopolistic supplier sells to two risk neutral customers \( (i = 1, 2) \). Each customer can be one of two
unknown types. A “high” type customer enjoys higher marginal utility from the good than a “low” type customer. Specifically, customer $i$ receives total utility $u_i = v^i D_i^s - m_i$ if she is of type $t_i \in \{l, h\}$. $v^h > v^l$ represents the marginal value for each type, $D_i^s$ is customer $i$’s demand being satisfied by the supplier, and $m_i$ is the total monetary transfer customer $i$ pays to the supplier.

Each customer knows her own type $t_i$ but does not know for certain the other’s type. The supplier cannot observe the customers’ types either. The common belief is that $t_i = h$ with probability $\lambda \in (0, 1)$ and $t_i = l$ with probability $1 - \lambda$. The realizations of $t_1$ and $t_2$ are independent.

Each customer demand, $D_i$, is uncertain and could be either $D_i^l$ (with probability $\alpha$) or $D_i^H$. The realization of $D_i$ is independent of customers’ type realization, $t_i$. We denote $D = (D_1, D_2)$ as the demand vector and $D = D_1 + D_2$ as the aggregate demand. In addition, we assume that $D^H < 2D^l$. After $D_i$ is realized, each customer decides how much of the demand should be submitted to the supplier, which we denote as $D_i^e$. In this model, we restrict $D_i^e \leq D_i$ implying that the customer cannot submit a demand higher than the realization of their actual demand. This restriction makes sense when the demand can be verified ex post. Taking the example of network traffic management, customer sends out files of certain sizes. A customer can increase the demand by extending the size of the file. However, she cannot benefit from doing so. Failure to impose this restriction introduces customers’ strategic behavior when submitting their demand. Cachon and Lariviere (1999) analyzed this kind of strategic behavior under different allocation rules. They show that customers’ order inflation could be an equilibrium strategy and the supplier is worse-off due to the concern of such strategic behavior. However, this is not the focus of our paper.

To serve the customer demand, the supplier has to invest a certain level of capacity $K$ before the demand $D$ is realized. The marginal cost of capacity investment is $c_0$. Observing the customers’ submitted demand $D^s = (D_1^s, D_2^s)$, the supplier decides how much demand to satisfy for each customer, $D^e = (D_1^e, D_2^e)$, to maximize his expected revenue. The total amount of satisfied demand, $D^e = D_1^e + D_2^e$ is constrained by the supplier’s capacity $K$. In addition, we assume that $c_0 > \min((2\alpha - \alpha^2)v^l, \alpha v^h)\lambda$ so that the capacity cost is high enough that the supplier will never choose to simply invest in full capacity. We also assume that $c_0 < (2\alpha - \alpha^2)v^h$. This condition guarantees that the high type customers’ willingness to pay is high enough so that the efficient capacity level is higher if both customers are high type than that if at least one customer is low type. $c_0 > (2\lambda - \lambda^2)v^h$ is imposed to guarantee that the chance that a customer is of high type is not big enough for the supplier to only serve the high type customers. Assuming the supplier is risk neutral and there is no cost for executing the customers’ demand, the potential supplier profit $\Pi$ is calculated as

$$\Pi = m_1 + m_2 - c_0 K.$$

We propose that the supplier can use option contracts to manage congestion (i.e., in the event that $D > K$). When capacity is insufficient to meet the aggregate demand, customers can choose to exercise their options, guaranteeing that their demand be executed. One unit of the option contract guarantees the customer one unit of satisfied demand regardless of congestion. To implement price discrimination, the options are priced such that only high type customers will buy the options because they suffer more from demand loss than low types. By using the options, high type customers can avoid demand loss but may pay a higher price. In addition, low type customers’ demand will be executed at a lower priority, increasing their potential demand loss. The supplier benefits from using the option contracts.
since he can, in effect, charge the high types a higher fee and is able to adjust capacity after observing customers’ actual types. From a social optimal perspective, the allocation efficiency is always guaranteed since those demands with higher marginal value always receive first priority execution.

We use two benchmark models for comparison with the options framework. In the first benchmark model, the supplier does not have the ability to distinguish customers’ types. He invests in capacity before the demand uncertainty is resolved and can only charge a linear price, \( p \), for each unit of executed demand. The second benchmark model is a full information case, in which the supplier can observe the customers’ types before the capacity investment. The supplier charges different prices \( p^H \) and \( p^L \) to the high types and low types, respectively. In this case, allocation efficiency is guaranteed since the supplier always satisfies demand from high type customers first and uses the remaining capacity (if there is any) to serve the low type customers. In addition, the supplier can set the prices that leave no surplus to both types of customers and the capacity investment will be efficient. We will examine both the supplier’s expected profit \( E\Pi \) and the efficiency \( W \)—defined as the sum of the supplier’s expected profit and the expected utilities of both customers—in these two benchmark models. We then compare them to our options framework.

### 2.1. Benchmark I—No Discrimination Case

In this section, we examine the case where the supplier can only charge a single linear price \( p \) to the customers regardless of their types. From the definition of \( D^* \), we have that \( D^* = \min\{K, D^1\} \). Each customer is charged \( m_t = pD_t \) and the supplier’s profit is \( \Pi(p, K) = pD^* - c_0K \). In this case, we have

\[
D^*(p) = \begin{cases} 
D_1 & \text{if } p \leq v^L \\
0 & \text{otherwise}
\end{cases}
\]

By charging a price higher than \( v^L \), the supplier only serves the high type customers. The supplier can enjoy a higher marginal profit from each unit of demand, but the supplier loses business if a customer is of low type. Proposition 2.1 states that the supplier should serve both types of customers by charging a price \( p = v^L \) when \( c_0 > (2\lambda - \lambda_\nu)v^L \). It also determines the optimal capacity decision \( K^{ND} \), the supplier’s expected profit \( E\Pi^{ND} \), and the overall efficiency \( W^{ND} = E(\Pi^{ND} + u_1^{ND} + u_2^{ND}) \). The superscript \( ND \) represents the no discrimination case.

**Proposition 2.1.** Assume the supplier can only charge a unit price, \( p \). The optimal price \( p^{ND} = v^L \) and the capacity \( K^{ND} = 2D^L \). Both types of customers will submit their demand and the supplier’s profit is

\[
E\Pi^{ND} = (v^L - c_0)2D^L.
\]

The overall efficiency is

\[
W^{ND} = (\lambda v^H + (1 - \lambda)v^L - c_0)2D^L.
\]

**Note.** See appendix for all the proofs.

Proposition 2.1 provides a benchmark outcome where the supplier must charge all customers a single unit price after the capacity is invested. In this case, the capacity level is barely enough for the minimal level of aggregate demand, that is, \( K^* = 2D^L = \inf\{D\} \). The low type customers are left no surplus and the high type customers can make strictly positive surplus which is proportional to the difference in the marginal utility of these two types, that is \( v^H - v^L \).

### 2.2. Benchmark II—Full Information Case

We use superscript \( FI \) to indicate the “full information” case. If the supplier can distinguish the types of the customers before the capacity is invested and charge different prices according to types, the optimal prices will be \( p^H(t = h) = v^h \) and \( p^L(t = l) = v^L \). The supplier hence leaves no surplus to both types of customers. When \( D > K \), the supplier will satisfy the high type customers’ demand first since he can get a higher margin.

The supplier’s optimal capacity decision \( K^{FI} \) is determined based on the customers’ types \( (t_1, t_2) \). Due to symmetry, \( K^{FI}(h, l) = K^{FI}(l, h) \). Lemma 2.2 specifies this optimal capacity level.

**Lemma 2.2.** In the full information benchmark case, the capacity investment decision is made contingent on the actual types of the two customers.

\[
K^{FI}(t_1, t_2) = \begin{cases} 
D^H + D^L & \text{if } t_1 = t_2 = h \\
2D^L & \text{otherwise}
\end{cases}
\]

Since the supplier charges prices which leave both types of customers zero surplus, the optimal capacity level maximizes both the supplier’s expected profit \( E\Pi^{FI} \) and the overall efficiency \( W^{FI} \). Hence, \( K^{FI} \) represents the efficient capacity level.

**Proposition 2.3.** When the supplier can observe customer types before making a capacity investment and charges different prices accordingly, the expected supplier profit equals the overall efficiency. That is, \( E\Pi^{FI} = W^{FI} \). Comparing with \( E\Pi^{ND} \) and \( W^{ND} \), yields

\[
\begin{cases} 
E\Pi^{FI} = E\Pi^{ND} + \Delta_1 + \Delta_2 + \Delta_3 \\
W^{FI} = W^{ND} + \Delta_1 + \Delta_3
\end{cases}
\]

where:

\[
\begin{align*}
(1) \quad & \Delta_1 = 2\lambda(1 - \lambda)\alpha(D^H - D^L)(v^h - v^L), \\
(2) \quad & \Delta_2 = (v^h - v^L)2D^L \quad \text{and} \\
(3) \quad & \Delta_3 = \lambda^2(2\alpha - \alpha_\nu)(v^h - c_0)(D^H - D^L).
\end{align*}
\]

It can be shown that \( \Delta_1, \Delta_2, \) and \( \Delta_3 \) are all strictly positive. The results that \( E\Pi^{FI} > E\Pi^{ND} \) and \( W^{FI} > W^{ND} \) always hold. The supplier’s profit gain comes
in three parts. $\Delta_i$ represents the profit gain by prioritizing the customers’ demand. $\Delta_r$ represents the profit gain from charging different prices according to types. $\Delta_o$, which is proportional to $(D^H - D^L = \Delta^H(h, h) - \Delta^L$, represents the profit gain from the ability to change capacity after actual realization of the customer types. The efficiency gain comes in two parts because the supplier’s ability to charge the high type customers a higher price only affects the monetary transfer among parties; it does not change overall efficiency.

2.3. Option-Capacity Game

In this section, we discuss the framework in which the supplier offers buyers an option contract to hedge their risk of demand loss. The customers choose the number of options to buy before the capacity is built. The supplier will then observe the number of options purchased by each customer and decide how much capacity he should build to meet the demand. After the capacity is built and the demand is realized, customers observe the supplier’s capacity and the aggregate demand and decide how many options to exercise, if they have any. The number of options a customer chooses to exercise is called the option demand and is denoted as $D^o_i$. We assume that the customer cannot exercise more options than the actual demand, that is $D^o_i \leq D^r_i$.

To successfully discriminate the customers, the supplier must set the option contract parameters ($p_o$, $p_r$) in such a way that only high type customers will buy them. The option contract also sets a unit price $p = \nu^i$ for regular demand. If the aggregate option demand, $D^o \equiv D^o_1 + D^o_2$, exceeds the capacity $K$, the supplier needs to buy back some of the options. The option buy back price has to be high enough so that the high type customers are willing to sell it back. Meanwhile, it cannot be too high to make the customers want to sell all the options back instead of executing them. Thus, the buy-back price, $p_o$, should equal $\nu^o - p_r$, the marginal benefit the high type customers get from the demand being executed.

According to the time line in Figure 1, the strategic interactions among the supplier and the two customers can be described in a three-stage game. In the first stage, the supplier announces the option contract parameters, $p_o$ and $p_r$. The customers simultaneously decide the number of options, $O_i$, to buy based on their own types. We are interested in the equilibrium cases where $O^o_i(t = 1) = 0$ and $O^o_i(t = h) > 0$. With our assumption of the customer demand, the high type customers have actually two choices: whether to buy $O_i = D^L$ for a minimal hedge or to buy $O_i = D^H$ for a maximal hedge.

The supplier observes $(h, t)$ through the sale of the options and decides the capacity, $K(h, t)$, to maximize his expected payoff in the following period, $E\pi(K)$.

After the demand is realized, the customers simultaneously decide how much demand to submit to the supplier, denoted as $D^r_i$, and how much of $D^r_i$ is submitted as option demand, $D^o_i$. When the regular price $p = \nu^i$, $D^r_i = D^o_i$ hold for both types of customers. The supplier gathers the total demand ($D^o_i$, $D^r_i$) and decides how to allocate the constrained capacity with the priority of the option demand. The total charge to a customer will be

$$m_i = p_o O_i + p_r D^r_i + \nu^i (D^o_i - D^r_i)^+ - \nu^o (D^o_i - D^r_i)^+,$$

where the last term is the customer’s expected compensation if the option demand is not executed. For the low type customers, since they won’t buy any option in our equilibrium, we can simplify the total charge as $m_i(t = 1) = \nu^r D^r_i$. The customer’s overall utility:

$$u_i = \begin{cases} (\nu^o - p_r) D^r_i + (\nu^o - \nu^i)(D^o_i - D^r_i)^+ - p_o O_i & \text{if } t = h \\ 0 & \text{if } t = 1 \end{cases}$$

In the following subsections, we use backward induction to resolve the three-stage game.

2.3.1. The Consumption Period. In this period, each customer observes their demand, which could be either $D^H$ or $D^L$. After observing the aggregate demand, $D$, and the capacity, $K$, the two customers simultaneously decide how many options to exercise.

Denote $O \equiv (O_1, O_2)$. There are six configurations that need to be discussed: $O = (D^H, D^H), (D^H, 0), (D^L, 0), (D^L, D^H), (0, 0)$ and $(D^H, D^L)$. Due to symmetry, we do not need to analyze the cases of $(D^L, D^L), (0, D^H)$ and $(0, D^L)$. On the equilibrium path, the only high type customers will buy options. Therefore, supplier will infer that both customers are of high type when $O = (D^H, D^H)$. Denote $\Phi = \min(0, D^r_i)$. The customer’s overall utility:

$$u_i = \begin{cases} (\nu^o - p_r) D^r_i + (\nu^o - \nu^i)(D^o_i - D^r_i)^+ - p_o O_i & \text{if } t = h \\ 0 & \text{if } t = 1 \end{cases}$$

maximize

$$\max_{D^o \leq \min(0, D^r_i)} (\nu^o - p_r) D^r_i + (\nu^o - \nu^i)(D^o_i - D^r_i)^+$$

subject to

$$\Phi = \min \left\{ (K - D^o_i, D^o_i)^+ \div (D^o_i - D^r_i)^+ \right\}$$

where $D^o_i$ is the option demand submitted by the other customer. $D^o_i = 0$ if that other customer is of low type. $\Phi$ indicates the probability that regular demand is satisfied. $\Phi = 1$ when $D < K$ and $\Phi = 0$ when $D^o_i > K$.

**Lemma 2.4.** Denoting $\mathcal{O}^o$ as the solution set of the above maximization problem, we have $\mathcal{O}^o \subseteq [0, \min(D^o_i, O)]$.

Lemma 2.4 states that a high type customer will either exercise all the available options to execute the demand or not exercise the options at all. When $p_r$ increases, the customer pays more to exercise the options and hence is increasingly reluctant to use them.
### Table 1  Option Demand When $\Omega = (D^h, D^l)$

<table>
<thead>
<tr>
<th>$\Omega = (D^h, D^l)$</th>
<th>$\mathbb{D} = (D^h, D^l)$</th>
<th>$D^h$</th>
<th>$D^l$</th>
<th>$D^h$</th>
<th>$D^l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v^h - (v^h - v^l) \min {k/2D^l, 1} &lt; p_o \leq v^l$</td>
<td>$D^h = 0$</td>
<td>$D^l = 0$</td>
<td>$D^h = 0$</td>
<td>$D^l = 0$</td>
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<tr>
<td>$v^h - (v^h - v^l) \min {k/2D^l + D^l, 1} &lt; p_o \leq v^h - (v^h - v^l) \min {k/2D^l, 1}$</td>
<td>$D^h = D^l$</td>
<td>$D^l = 0$</td>
<td>$D^h = 0$</td>
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<tr>
<td>$v^h - (v^h - v^l) \min {k/2D^l, 1} &lt; p_o \leq v^h - (v^h - v^l) \min {k/2D^l + D^l, 1}$</td>
<td>$D^h = D^l$</td>
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<tr>
<td>$p_o \leq v^h - (v^h - v^l) \min {k/2D^l, 1}$</td>
<td>$D^h = D^l$</td>
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Tables 1 through 6 show the solutions of $D^h_o$ as functions of different realizations of $\Omega$, $\mathbb{D}$, $p_o$, and $K$.

The results in Tables 1 through 6 show that the amount of options a high type customer will exercise decreases as the option strike price $p_o$ increases. Moreover, we can see that when $p_o \geq v^h - (v^h - v^l)(k/2D^l)$, no option will be exercised in any configuration of $\Omega$ and demand realization $\mathbb{D}$. This is because that the options are too expensive to exercise. Therefore, the options have no value. In the following discussion, we focus on the cases where the strike price $p_o < v^h - (v^h - v^l)(k/2D^l)$.

#### 2.3.2. Capacity Investment Game

In the above section, we analyze both customers’ equilibrium decisions on $D^h_o$. This decision is contingent on the number of options both customers have purchased $\Omega$, the realized demand $\mathbb{D}$, the option strike price $p_o$, and the capacity $K$. In this section, we analyze the supplier’s optimal capacity decision $K^*$, which is made before the actual demand is realized. The optimal capacity decision $K^*$ is hence based on the supplier’s observation of option purchase by the customers $\Omega$ and the strike price $p_o$, while expecting the possible future demand realization $\mathbb{D}$ and the customers’ reaction to $D^h_o$, $i = 1, 2$.

Once the customers purchase the options, the supplier will only look at the future revenue excluding the revenue from option purchase, $p_o(O_1 + O_2)$, to decide the optimal capacity. When $K^*$ increases, the chance of congestion decreases. The supplier’s revenue comes more from serving the regular demand. When $p_o < v^l$, it is more profitable for the supplier to increase capacity. However, if $p_o > v^l$, the supplier tends to increase the customers into exercising their options by restricting the capacity $K$. This may create problems when $K < D^l$ because the supplier fails to satisfy the option demand as they promised in the option contract. He must compensate customers whose option demands cannot be satisfied.

Customers are more willing to exercise their options as $p_o$ decreases. Forseeing this, the supplier tends to increase the capacity level to guarantee all the exercised options can be satisfied. This will minimize the expected compensation. Hence, we predict capacity $K^*$ decreases as $p_o$ increases.

**Proposition 2.5.** In a subgame perfect equilibrium, the supplier’s optimal capacity decision is as follows:

1. When $\Omega = (D^h, D^l)$, the optimal capacity $K^* = \left\{ \begin{array}{ll}
D^h + D^l & \text{if } v^h - (v^h - v^l) \frac{D^l}{D^h} < p_o \leq v^h \\
v^l - p_o \cdot \frac{D^h}{2D^l} & \text{if } v^h - (v^h - v^l) \frac{D^h + D^l}{2D^l} < p_o \leq v^h - (v^h - v^l) \frac{D^l}{D^h} \\
2D^l & \text{if } p_o \leq v^h - (v^h - v^l) \frac{D^h + D^l}{2D^l}.
\end{array} \right.$

2. When $\Omega = (D^h, D^l)$, the optimal capacity $K^* = \left\{ \begin{array}{ll}
D^h + D^l & \text{if } v^h - (v^h - v^l) \frac{2D^l}{D^h + D^l} < p_o \leq v^h \\
v^l - p_o \cdot (D^h + D^l) & \text{if } v^l < p_o \leq v^h - (v^h - v^l) \frac{2D^l}{D^h + D^l} \\
2D^l & \text{if } p_o \leq v^l.
\end{array} \right.$

3. When $\Omega \in \{(D^l, D^l), (D^h, 0), (D^l, 0), (0, 0)\}$, the optimal capacity $K^* = 2D^l$ for all $p_o < v^h$.  

_**Fang and Whinston:** Option Contracts and Capacity Management_  
Table 2  Option Demand When $\Omega = (D^b, D^d)$

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<tr>
<th>$\Omega = (D^b, D^d)$</th>
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<tr>
<td>$v^b - (v^b - v^d)\min{K/2D^b, 1} &lt; \rho_x \leq v^b$</td>
<td>$D_2^b = 0$</td>
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<tr>
<td>$v^b - (v^b - v^d)\min{K/2D^b + D^d, 1} &lt; \rho_x \leq v^b - (v^b - v^d)\min{K/2D^b, 1}$</td>
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<td>$v^b - (v^b - v^d)\min{K/2D^b + D^d, 1} &lt; \rho_x \leq v^b - (v^b - v^d)\min{K/2D^b, 1}$</td>
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Table 3  Option Demand When $\Omega = (D^b, 0)$

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Table 4  Option Demand When $\Omega = (D^d, D^d)$

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<tr>
<td>$v^b - (v^b - v^d)\min{(K + 2(D^d - D^b))(D^d + 2(D^d - D^b)), 1} &lt; \rho_x \leq v^b$</td>
<td>$D_2^d = 0$</td>
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<tr>
<td>$v^b - (v^b - v^d)\min{(K/2D^d + D^d)(D^d + 2(D^d - D^b)), 1} &lt; \rho_x \leq v^b$</td>
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Table 5  Option Demand When $\Omega = (D^d, 0)$

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Table 6  Optimal Demand When $\Omega = (0, 0)$

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Figure 2 summarizes the optimal capacity level $K^*$ as a function of $\rho_x$ in all six configurations of $\Omega$. The capacity is at least the minimal level of the aggregate demand $2D^d$. It is greater than $2D^d$ only when both customers have purchased the options and at least one
of them bought $D^H$ options. In all other cases, we have $O_1 + O_2 < 2D^L$ and hence the total number of options demanded, which is smaller than the total number of options purchased, must be smaller than $2D^L$. As a result, the supplier will remain unconcerned about the compensation even when he sets the capacity $K = 2D^L$.

However, when both have purchased options and at least one bought $D^H$, the supplier has to worry about the situation that both customers exercise all their options and the capacity may be insufficient for the option demand. If he stays with the capacity level $K = 2D^L$, the probability of providing compensation will be as high as $2/2$ when $p_e$ is low enough. So our assumption that $(2/2) > 0$ suggests that the supplier should increase the capacity from $2D^L$ to avoid the situation. The assumption that $(2/2) < 0$ suggests that the supplier cannot be better off by increasing the capacity from $D^H$ to avoid the possible compensation when $(D^H, D^H)$. The optimal capacity is between $2D^L$ and $D^H + D^L$.

2.3.3. Optimal Option Prices. Given the optimal decisions of $K^*$ and $(D^H_1, D^H_2)$, we can now analyze the first stage of the option-capacity game to derive the optimal option contract $(p_o, p_e)$ and the supplier’s expected profit by using the option contracts. All the decisions analyzed in Sections 2.3.1 and 2.3.2 are contingent on both the option strike price $p_e$ and the customer’s option purchase $O$. In addition, the customers decide $O$ based on their own types $(t_1, t_2)$ and how the supplier prices the options. The fundamental question to be resolved is: what is the optimal $p_o$ and $p_e$ that induces the customer to purchase the right number of options and maximizes supplier profit?

In this period, the supplier announces the option prices $(p_o, p_e)$. Then the customers, who do not know each other’s type, simultaneously submit their option purchase demand $O_i$ to the supplier. The option is priced such that the low type customers will not feel profitable purchasing the options but the high type customers will.

The customer’s incentive of buying options is different when the other customer’s option purchase decision, $O_{-i}$, varies. If we denote the expected value of an option as $f_o(O_i, O_{-i})$, $f_o$ can be calculated as follows:

$$f_o(O_i, 0) = 1/O_i \{ Eu_h(O_i, O_{-i}) - Eu_h(0, O_{-i}) \}$$

where $Eu_h(O_i, O_{-i})$ represents the expected utility a high type agent gets after purchasing $O_i$ units of options while the other customer has purchased $O_{-i}$ units of options.

**Lemma 2.6.** $f_o(O_i, D^H) \geq f_o(O_i, D^L) \geq f_o(O_i, 0)$.

Lemma 2.6 states the fact that the option is more valuable to a high type customer if the other customer buys more options. This is so because the two customers will compete for the limited capacity resource in
the consumption period. Buying more options protects them in the competition. Consequently, if the supplier can induce one customer to buy $D^{H}$ units of options, the other customer would want to pay more for the options if she is a high type. This implies that the supplier prefers the equilibrium where the high type customers choose the maximal hedging strategy, as shown in Proposition 2.7.

**Proposition 2.7.** The supplier maximizes his expected profit when $p^{*} = v^{h} - (v^{h} - v)(D^{H} + D^{L})/2D^{H}$ and $p_{o}^{*} = \lambda f(D^{H}, D^{H}) + (1 - \lambda)f(D^{H}, 0)$. In such an equilibrium, a high type customer will choose a maximal hedging strategy $O_{i} = D^{H}$. The expected supplier profit

$$E\Pi^{*} = (v^{l} - c_{o})2D^{L} + 2\alpha\lambda(v^{h} - v^{l})(D^{H} - D^{L}) + \lambda^{2}(2\alpha - \alpha^{2})v^{h} - c_{o})(D^{H} - D^{L})$$

and the equilibrium capacity investment

$$K^{*} = \begin{cases} D^{H} + D^{L} & \text{if } O_{1} = O_{2} = D^{H} \\ 2D^{L} & \text{otherwise.} \end{cases}$$

The supplier’s capacity investment $K^{*}$ is the same as the one in the full information benchmark case (see Section 2.2) given the belief that $O_{1}(t = h) = D^{H}$ and $O_{2}(t = l) = 0$. By using option contracts, the supplier can make a contingent capacity investment based on the customer’s option purchase decision $O_{i}$, which reveals the customer types $(t_{1}, t_{2})$ in equilibrium. This flexibility improves the capacity investment decision. The option compensation $p_{o} = v^{h} - p_{r}$ is important in achieving the efficient capacity level $K^{*} = K^{FL}$. Under the options framework, the supplier’s incentive to increase the capacity when both customers are of high type is to satisfy as many options as possible while reducing the compensation. That is, the marginal benefit the supplier gets is $p_{r} + p_{o} = v^{h}$ when $K < D^{o}$. This incentive is well-aligned with the full information benchmark case where the incentive of increasing capacity is to increase the ability of serving high type demand, which yields a marginal profit $p^{H}(t = h) = v^{h}$.

We can rewrite the supplier’s expected profit as:

$$E\Pi^{*} = E\Pi^{ND} + \Delta_{1} + \Delta_{3} + \Delta_{4}$$

where $\Delta_{1}$ and $\Delta_{3}$ are defined in Proposition 2.3. $\Delta_{1}$ represents the gain from prioritizing the high type customer’s demand over the low type one’s. $\Delta_{3}$ refers to the gain achieved when the supplier increases capacity after observing two high type customers. $\Delta_{4} = 2\alpha\lambda^{2}(v^{h} - v)(D^{H} - D^{L}) > 0$. $a(v^{h} - v)(D^{H} - D^{L})$ is the expected loss if a high type customer suffers if she doesn’t buy options but competing for capacity with another high type customer who has options. $\Delta_{4} = 2\lambda^{2}$ times this expected loss representing the supplier’s expected gain from the competition between two high type customers.

Comparing $E\Pi^{*}$ to $E\Pi^{FL}$, we can characterize supplier’s profit loss when he cannot distinguish the high type customer and extract full surplus from them as follows:

$$E\Pi^{FL} - E\Pi^{*} = \Delta_{2} - \Delta_{4} = 2\alpha\lambda eu_{0}(0, D^{H}) + (1 - \lambda) eu_{0}(0, 0)$$

$(\lambda eu_{0}(0, D^{H}) + (1 - \lambda) eu_{0}(0, 0))$ is the reserve utility a high type customer has when she does not buy options. This reserve utility is also known as the Information Rent in the price discrimination literature, representing the cost the supplier pays to induce a high type customer to reveal her type. The expected information rent the supplier pays is exactly $\Delta_{2} - \Delta_{4}$, which increases as probability $\lambda$ increases.

**Proposition 2.8.** $W^{*} = W^{FL}$.

The capacity investment is efficient in equilibrium and high type customers will always be served as the first priority. Hence, it is not surprising that the option contracts can achieve the same efficiency level as the full information benchmark case. Proposition 2.8 justifies the optimality of our proposed option framework.

3. Discussion and Extensions

3.1. Using Options for Price Discrimination

In the literature of price discrimination under unobservable types, the supplier’s ability to employ a nonlinear pricing scheme (e.g., quantity discount and bundling) is critical to his profit. In this paper, we show that with properly priced options, the supplier can achieve the same profit level with a simple linear pricing function using an option framework. Our framework also works when the customer’s demand is uncertain.

In our framework, a customer faces two purchase decisions. She chooses the number of options to purchase before the demand realization and the amount of demand (including the numbers of regular demand and option demand) to request afterwards. In both decisions, she has full flexibility to choose the quantities. A linear pricing scheme is applied in all the purchases. The customers prefer such flexibility when they suffer from demand uncertainty.

To discriminate the customers, we pay attention to the case where only high type customers will buy the options. A high type customer’s total charge in equilibrium can be divided into two parts. A fixed payment $p_{o}D^{H}$ is charged ex ante regardless of her actual demand realization to guarantee the priority of her demand execution. In addition, she pays a contingent payment based on her actual demand realization. Adjusting the option price $p_{r}$ and $p_{o}$, the supplier essen-
tially changes the ratio between the ex ante and ex post payments to affect the high type customers hedging incentive and exploit their willingness to pay for the demand.

The option framework helps the supplier to conduct price discrimination when the customer’s demand is uncertain. When there is no demand uncertainty (i.e., $D^H = D^L$), the capacity will always be enough for the aggregate demand under our assumption. Hence, the option has no value and the supplier cannot discriminate among the customers.

Our assumption of the supplier’s marginal capacity cost is critical to derive the result. The discrimination framework is built based on the high type customer’s concern of potential demand loss. If the capacity cost is too small, the customers can infer that the supplier will always build enough capacity for the demand and won’t pay money ex ante to hedge the potential demand loss. The discrimination is not implementable in this case. If the capacity is too large, the supplier will find it profitable to charge a high regular price (e.g., $v^H$) to exploit the high type customers’ surplus only.

### 3.2. Multiple Agents and Multiple Types

In this paper, we use a parsimonious model with one supplier and two customers to illustrate price discrimination. The model can be extended to a case where multiple customers with two possible types. In such a model, we could still design the option contracts so that only those high type customers will buy them. The major difference would be a more complicated aggregate demand pattern. In a symmetric equilibrium where all high type customers adopt the same strategy, the supplier can separate the customers into two groups and treat them as two representative agents. The same analysis can then be applied to figure out the optimal option contract. The efficiency level of the second degree price discrimination can still be achieved.

The model gets further complicated with multiple type customers. The supplier could design multiple option contracts with different combinations of strike prices $p_c$ and compensation $p_b$ for each type. The customers could then self-select the options and decide when to exercise them. The demand should then be prioritized according to the supplier’s marginal punishment of not fulfilling the demand (i.e., $p_b - p_c$). As a result, different demand associated with different option contracts are categorized into different priority levels. Allocation efficiency can be achieved if the customers with higher willingness to pay will always buy the option contracts with higher priority (characterized by $p_b - p_c$). The challenge is how to price the options to induce the customers to purchase the right options to reveal their types.

### 3.3. Spot Exchange Options Market

In our setup, customers purchase options to hedge their demand risk. After their demand is realized, they can decide how many of their options to exercise. In this setup, a customer cannot buy additional options from other customer ex post. This raises a question: what if they can exchange their options after observing their demand?

On the one hand, allowing to exchange options leads to more efficient option utilization. If a customer is able to sell her extra options after the demand is realized, she is more willing to buy options ex ante. Thus, the ex post exchange encourages the option purchase ex ante. On the other hand, the customers’ incentive for a maximal hedge decreases with the possibility of option exchange. This is because she may find someone who will sell options to her if her demand is high. Between this conflicting incentives, it is not clear which incentive is stronger in general.

However, if we assume that customer types may change ex post, then the existence of an option exchange market helps in some cases. A detailed analysis of when a spot exchange market helps increase a monopolist’s profit can be found in Geng, Wu, and Whinston (2007). Moreover, ex post exchange reduces the customer’s ex post risk of purchasing options. Hence, the existence of an ex post exchange market always improves the option purchase incentive if the customers are risk-averse.

### 4. Conclusion

In this paper, we propose an option framework that allows a monopolistic supplier to conduct price discrimination among customers, thereby maximizing his expected revenue under demand uncertainty and a capacity constraint. Our analysis shows that option contracts can benefit the supplier because the high type customers will pay more for hedging potential demand loss. The supplier gains the additional benefit of being able to adjust capacity according to his observation of customer types.

We also analyze the strategic interactions among the supplier and customers. We show that, in equilibrium, the efficient capacity level can be induced by setting a compensation price which leaves the high type customers indifferent about whether to exercise their options or to ask for compensation. Overall efficiency is guaranteed and the supplier and the high type customers share the efficiency gain from the efficient capacity investment.

Our proposed structure replicates the classical price discrimination outcome where the low type customers do not gain surplus and the high type customers enjoy an information rent. Our proposed structure can easily be adopted in situations where the supplier is not...
 allowed to sell a bundled product with fixed quantity and situations where the actual demand and capacity is not contractible. Our framework has significant revenue management implications for various industrial applications such as network capacity management, airline ticket reservation, and telephone and electricity providers.

Acknowledgment
This work was funded in part by NSF grant No. IIS-0219825.

Appendix

Proof of Proposition 2.1
Proof. If \( p \leq \bar{v} \), both customers submit all their demand to the supplier,

\[
D_i = D = \begin{cases} 
D^H & \text{with } pr = \alpha \\
D^L & \text{with } pr = 1 - \alpha.
\end{cases}
\]

The supplier’s expected profit

\[
EII = p[\alpha^2 \min \{K, 2D^H\} + 2(1 - \alpha) \min \{K, D^H + D^L\} + (1 - \alpha)^2 \min \{K, 2D^L\}\] - c_0 K.
\]

Maximizing the expected profit under the condition \( p \leq \bar{v} \), we have \( p^* = \bar{v} \) and \( K^* = 2D^L \). The supplier’s expected profit is \( EII(p = \bar{v}) = (\bar{v} - c_0)2D^L \). Customer’s expected utility is \( u_i = (\bar{v}_i - \bar{v})D^L \).

If \( p \in (\bar{v}, \bar{v}^p) \), only high type customers will submit the demand. \( D^H = D \) when \( t_1 = h \) and \( D^L = 0 \) for \( t_1 = l \).

When \( \bar{v}^p < p \leq \bar{v}^h \), the supplier’s expected profit

\[
EII(p) = \lambda^2 p[\alpha^2 \min \{K, 2D^H\} + 2(1 - \alpha) \min \{K, D^H + D^L\} + (1 - \alpha)^2 \min \{K, 2D^L\}\] + 2(1 - \alpha)^2 \min \{K, D^L\}\] + 2(1 - \alpha) \min \{K, D^H\}\] - c_0 K.
\]

Maximizing the expected profit and applying the assumption \( 2(1 - \lambda)\bar{v}^p < c_0 \), we have \( p^* = \bar{v}^p \) and \( K^* = 0 \). Thereby, \( EII(p = \bar{v}^p) = 0 \) and it is never profitable to serve high type customers only.

In summary, we conclude that the supplier’s best strategy is always to set \( p^{ND} = \bar{v} \) to serve both types of customers.

The optimal capacity will be \( K^{ND} = 2D^L \). The expected profit \( EII^{ND} = (\bar{v} - c_0)2D^L \) and the overall efficiency:

\[
W^{ND} = EII^{ND} + \lambda E\eta^{ND}(t_1 = h) + (1 - \lambda) E\eta^{ND}(t_1 = l) = [\lambda \bar{v}^p + (1 - \lambda) \bar{v}^L - c_0]2D^L.
\]

\( \square \)

Proof of Lemma 2.2
Proof. The optimal capacity \( K^{FI} \) is made contingent on the customer types \( T = (t_1, t_2) \). When \( T = (h, h) \), the supplier’s expected profit

\[
EII^{FI}(K, T) = \bar{v}^h[\alpha^2 \min \{K, 2D^H\} + 2(1 - \alpha) \min \{K, D^H + D^L\} + (1 - \alpha)^2 \min \{K, 2D^L\}\] - c_0 K.
\]

which is maximized when \( K(h, h) = D^H + D^L \) from the assumption \( \alpha\bar{v}^h < c_0 < (2\alpha - \alpha^2)v^p \).

When \( T = (l, l) \), similarly, we can have \( K^{FI}(l, l) = 2D^L \), since \( (2\alpha - \alpha^2)v^L < c_0 < \bar{v}^L \).

When \( T = (h, l) \) or \((l, h) \) and \( K > D^H \), the supplier’s expected profit

\[
EII^{FI}(K, (h, l)) = \alpha(\bar{v}^L D^H + \bar{v}^L \min \{K - D^H, D^L\}) + \alpha(1 - \alpha)(\bar{v}^L D^H + \bar{v}^L \min \{K - D^H, D^L\}) + \alpha(1 - \alpha)(\bar{v}^H D^L + \bar{v}^L \min \{K - D^H, D^L\}) + (1 - \alpha)^2(\bar{v}^H D^L + \bar{v}^L \min \{K - D^H, D^L\}) - c_0 K.
\]

which is maximized when \( K^* = 2D^L \). It can also be shown that \( K \leq D^H \) cannot be optimal. Therefore, \( K^{FI}(h, l) = K^{FI}(l, h) = 2D^L \).

\( \square \)

Proof of Proposition 2.3
Proof. For the supplier, the probability that both customers are high types is \( \lambda^2 \). The expected profit

\[
EII(h, h) = (\bar{v}^L - c_0)2D^L + (\bar{v}^H - \bar{v}^L)2D^L + (2\alpha - \alpha^2)\bar{v}^L - c_0(D^H - D^L).
\]

With probability \( 2\lambda(1 - \lambda) \), one customer is of high type and the other is of low type. The expected profit \( EII(h, l) = (\bar{v}^L - c_0)2D^L + (\bar{v}^H - \bar{v}^L)(\lambda D^H + (1 - \alpha) D^L) \). With probability \( 1 - \lambda^2 \), both customers are of low type. The expected profit \( EII(l, l) = (\bar{v}^L - c_0)2D^L \). Since \( u_i(t_i = h) = u_i(t_i = l) = 0 \),

\[
W^{FI} = EII^{FI} + \lambda EII(h, h) + 2\lambda(1 - \lambda) EII(h, l) + (1 - \lambda)^2 EII(l, l) = (\bar{v}^L - c_0)2D^L + (\bar{v}^H - \bar{v}^L)2\alpha(1 - \lambda) \]
\(
\times (D^H - D^L) + (\bar{v}^H - \bar{v}^L)\lambda(2\alpha - \alpha^2)\bar{v}^L - c_0 \]
\(
\times (D^H - D^L) \)
\]
\( \square \)

Sketch Proof of Lemma 2.4
Proof. The proof is straightforward since we can show that the second order condition of the above objective function is non-negative.

\( \square \)

Sketch Proof of Proposition 2.5
Proof. Applying the outcomes from Table 1–6, we can derive the optimal capacity directly.

\( \square \)

Sketch Proof of Lemma 2.6
Proof. From the result of

(1) When \( p_r \geq V^L - (V^L - V^H)(D^H/D^L) \): \( K^* = 2D^L \) for all configurations of \( O \). No options will be exercised for realized \( D^L \). Hence, the option has no value, it is straightforward to conclude that \( p_r = 0 \);

(2) When \( V^L - (V^L - V^H)(D^H/D^L - D^L) \leq p_r < V^L - (V^L - V^L)(D^L/D^H) \): if the agent has bought \( O = D^L \), she will never exercise the options no matter what the other type of customer is. Hence, she would not pay for any positive \( p_r \) to purchase the option;

(3) When \( V^L - (V^L - V^H)(2D^L/D^H + D^L) \leq p_r < V^L - (V^L - V^L)(2D^L/D^H + D^L) \): if the agent has bought \( O = D^L \), she will never exercise the options no matter what the other type of customer is. Hence, she would not pay for any positive \( p_r \) to purchase the option.
We need to discuss the profit according to the different utility from the options are

\( \alpha_i = D^i; \) if the agent has bought \( O_i \)

\( \alpha_i = D^i; \) she will only exercise it when \( D = (D^i, D^j) \) and

\( D^\alpha_i \neq D^H. \) Hence if the other agent is of type I, she is

always better off by purchasing \( O_{-i} = D^H \) than \( O_{-i} = D^L. \) However, given \( O_{-i} = D^H, \) agent \( i \) will never

exercise her options and the value of the options is 0.

If the other agent is of type II, the agent’s expected utility from the options are

\[ \begin{align*}
\alpha_i = D^i; & \quad u_i(D^i) = \alpha_i^2(V^i - p_i D^i + (1 - \alpha^2)D^i) \\
\alpha_i = D^j; & \quad u_i(D^j) = \alpha_i^2((V^i - p_i)D^i + (V^i - V^j)(D^jD^j/(2D^j - D^i))) + (1 - \alpha^2)D^j - p_i D^j; \text{ and} \\
\alpha_i = 0; & \quad u_i(0) = (V^i - V^j)D^i.
\end{align*} \]

We can show that \( (u_i(D^i) - u_i(0)) + p_i(D^i) > 0 \)

and

\( (V^i - V^j)(D^j/D^i) > 0, \)

meaning that the agent is better off by purchasing \( O_i = D^i. \)

Following the same way, we can show that \( D^j \) is not the

optimal choice when \( p_x > V^i. \)

\[ \text{Sketch Proof of Proposition 2.7} \]

We need to discuss the profit according to the different \( p_x \) segments:

(1) \( p_x \geq V^i - (V^i - V^j)(D^i/D^j): \) no option is exercised, \( p_o = 0 \) and

\[ EII = (V^i - c_o)2D^i. \]

(2) \( V^i - (V^i - V^j)(D^i/D^j) \leq p_x < V^i - (V^i - V^j)(D^i/D^j), K^*(D^j, D^i) = ((V^i - p_x)/(V^i - V^j))2D^j. \]

\[ EII = E\pi + 2\lambda p_oD^i \]

\[ = \lambda^2 \left( (2\alpha - \alpha^2)V^i - c_o \right) \frac{V^i - p_x}{\sqrt{V^i - V^j}} 2D^j + (1 - \alpha^2)V^j2D^j \\
+ 2\lambda(1 - \lambda)(\alpha^2(p_x - V^i)D^j + (V^j - c_o)2D^j) \\
+ (1 - \lambda)^2(V^j - c_o)2D^j + 2\lambda(Eu_i(D^i)/Eu_i(0)) \]

which we can obtain \( dEII/dp_x < 0. \)

(3) \( V^i \leq p_x < V^i - (V^i - V^j)(D^i/D^j): \) \( K^*(D^j, D^i) = D^i + D^j, K^*(D^i, D^j) = K^*(0, 0) = 2D^j. \) One can get

\[ EII = (V^i - c_o)2D^i + 2\alpha\lambda(V^i - V^j)(D^j - D^i) \\
+ \lambda^2(2\alpha - \alpha^2)V^i - c_o)(D^i - D^j). \]

Here, \( dEII/dp_x = 0. \)

\[ \text{Sketch Proof of Proposition 2.8} \]

\[ \text{Proof.} \] The proof is straightforward from the proof of Proposition 2.7. \( \square \)

\[ \text{References} \]

Afeche, P. 2004. Incentive-Compatible Revenue Management in

Queueing Systems: Optimal Strategic Delay and other Delaying

Tactics. Working paper, Kellogg School of Management, North-

western University, Evanston, Illinois.

Birge, J. R. 2000. Option methods for incorporating risk into linear

capacity planning models. Manufacturing & Service Operation


Boyaci, T., S. Ray. 2006. The impact of capacity costs on product
differentiation in delivery time, delivery reliability, and prices.


Cachon, G. P., M. A. Lariviere. 1999. An equilibrium analysis of

linear, proportional and uniform allocation of scarce capacity.

IIE Transactions 31(9) 835–849.

Deshpande, V., L. Schwartz. 2002. Optimal Capacity Choice and

Allocation in Decentralized Supply Chains. Technical Report,

Krafft School of Management, Purdue University, West

Lafayette, Indiana.

Geng, X., R. Wu, A. B. Whinston. 2007. Profiting from Partial

Allowance of Ticket Resale. Journal of Marketing, April, 71.


to network computing with priority classes. Journal of Organizational

Computing and Electronic Commerce 6(1) 71–95.


Network Management. Communications of the ACM, September

42(9) 57–63.

Huchzermeier, A., M. A. Cohen. 1996. Valuing Operational Flexi-

bility under Exchange Rate Risk. Operations Research Jan-Feb

44(1) 100–113.


Demand Management. Management Science 49(8) 983–1002.


Noldeke, G., K. M. Schmidt. 1995. Option Contracts and Renegoti-

ation: A Solution to the Hold-up Problem. The RAND Journal of

Economics 26(2) 163–179.

Ozer, O., W. Wei. 2004. Inventory control with limited capacity and

advance demand information. Operations Research 52(6) 988–

1000.

Sodhi, M. S. 2004. Managing demand risk in tactical supply chain

planning. Production and Operations Management 14(1) 69–79.

Tirole, J. 1988. The Theory of Industrial Organization. MIT Press,

Cambridge, Massachusetts.