Study Questions: Chapter 5

Math 330: History of Mathematics

September 23, 2006.

Chapter 5: The Heroic Age

Here are some questions and comments related to your Chapter 5 reading. They should help you get ready for the quiz.

This book is written for an educated reader and uses advanced vocabulary. You should look up words that you do not know. I will periodically ask for volunteers to put new words up on the board, with definitions and etymologies, for some extra credit.

Centers of activity

1. Which century is this chapter about? (It is focused on the last half of this century)
2. Why does Boyer call this age the “Heroic Age of Mathematics”?
3. Name the seven individuals discussed in this chapter?

Anaxagoras of Clazomenae

1. What event started the 5th century BC, what event ended it, and what came between?
2. Why and where was Anaxagoras imprisoned?
3. Was Anaxagoras a contemporary to Pericles and Socrates?
4. According to Boyer, what was the typical Greek motive? [hint: the d to k]
5. Was Anaxagoras primarily a mathematicians? If not what was he? [hint: n p]
6. What problem did Anaxagoras work on when he was in prison?
7. Define this problem. This is the first of the three famous problems.
8. In the Greek world mathematics was closely related to what? [hint: ph]

Three Famous Problems

1. Did Anaxagoras’ life overlap with that of Plato?
2. What killed Pericles?
3. What did the oracle of Apollo at Delos tell the Greeks to do in order to avert the plague?
4. What happened to the alter’s volume when the sides were doubled in length?
5. What is another name for the “Delian problem”? Describe this problem.
6. What is the third famous problem.
7. In modern times, what was proven about these three problems?
8. Note: constructions with straightedge and compass were regarded by Greeks as solutions to a problem. However, some were happy to use other tools to solve these problems. This chapter gives the impression that the Greeks wouldn’t accept a solution to a problem that did not stick to the straightedge and compass, but that is an overstatement in my opinion.

Quadrature of Lunes

1. What was Hippocrates’ occupation before he turned to mathematics? [hint: me]
2. Note: he was involved in a long lawsuit or court case in Athens which gave him lots of spare time. In Athens he learned about the problem of squaring a circle, and became interested in geometry.
3. Did Hippocrates write a book called “Elements of Geometry”? What happened to it? Has it been translated into English?
4. Have any fragments of Hippocrates work survived? If so what did it deal with? [hint: the q of the l]
5. Squaring the circle was too hard for Hippocrates, so like any good mathematician, he decided to start with something easier. What did he try to square? [hint: the l]
6. What is a lune?
7. Was Hippocrates influenced by the Pythagoreans?
8. Hippocrates showed that the lune ABCD in figure 5.1 is equal in area to what triangle? Can one form a square with the same area?
9. In class we will discuss Hippocrates results in more detail. We will show that he could square some lunes but discuss the fact that he could not square all lunes. We will discuss how if one could square all lunes, then one could square the circle. Even squaring some lunes is a great achievement: it shows that sometimes curved regions can be squared.

Continued Proportions

1. Suppose that $a$ and $b$ are lengths. A mean proportional $x$ between $a$ and $b$ is a length such that $a : x$ equals $x : b$. Show that the rectangle with sides $a$ and $b$ has the same area as the square with side $x$. Explain why, if you could find $x$, then you could square a rectangle. Note: we will see how to find $x$ with straightedge and compass in class. Note also: the Greeks did not distinguish between a line segment and its length (they did not have real numbers) so I use the term length to refer to both.
2. What is the problem of finding two mean proportionals?
3. Hippocrates showed that the problem of finding two mean proportionals was related to which famous problem?

4. Explain (using fractions and algebra if you like) why finding two mean proportions between $a$ and $b = 2a$ solves the problem. Note: Greek mathematicians of this time did not use fractions, they used ratios instead. However, you may use fractions and algebraic manipulations to modernize Hippocrates’ argument.

5. Hippocrates did not solve but made genuine progress in two of the three famous problems. Which problem did he apparently not make progress in?

**Hippias of Elis**

1. Hippias of Elis was a Sophist. Were sophists allowed to earn money from sharing their knowledge?

2. Hippias was a contemporary of Socrates, and Plato wrote dialogues involving Socrates and his contemporaries. Based on Plato’s writings, describe Hippias’ personality. Warning: Socrates and his disciple Plato had a negative opinion of Sophists, and Plato’s writings were a mix of fact and fiction designed to make philosophical points. So we cannot rely on Plato to give us a fair portrait of Hippias.

3. Define the curve of Hippias. What are the names given to this curve? Challenge: try to write the curve in polar coordinates.

4. Given an acute angle, describe how to trisect it with Hippias’ curve.

5. Dinostratus, who came later, showed how to solve which problem with Hippias’ curve?

**Philolaus and Archytas of Tarentum**

1. Were Philolaus and Archytas both Pythagoreans?

2. What position did Archytas hold in Tarentum? Note: Tarentum is now called Taranto and has over 200,000 people. Which country is Taranto in today? (Actually it is the Romans that called it Tarentum, the Greeks who founded it called it Taras (Ταράς).)

3. What are some of the legends that suggest that Arcytas was basically a good guy?

4. What are the seven liberal arts? Which four did Archytas regard as mathematical and call the *quadrivium*?

**Duplication of the Cube**

1. Archytas solved the duplication of the cube problem. Did he do it with a ruler and straightedge?

2. Describe three solids that Archytas used to duplicate the cube. Note: I will go over this in more detail in class.

3. Was Archytas a friend or enemy of Plato?

4. Hippasus did something bad (from the Pythagorean point of view). What are three versions of what he did? What happened to Hippasus afterwards (two versions).
Incommensurability

1. The Pythagorean theory of proportion was based on the notion that every ratio was equivalent to a ratio of integers. So if I give you two magnitudes \( x \) and \( y \) of the same type, then I can find two integers \( m \) and \( n \) such that \( x : y \) equals \( m : n \). Their theorems for similar triangles may have been proved with that assumption. Also, their proof of the Pythagorean theorem may have been based on similar triangles and proportions.

   If two magnitudes have a ratio \( m : n \) where \( m \) and \( n \) are integers, then what are they called? [hint: c] What are they called if they do not have such a ratio? [hint: i]

2. Note: when incommensurable magnitudes were discovered it might have caused a crisis in the Pythagorean community. However, modern scholars have various opinions on this controversial claim. One thing is certain: the discovery of incommensurable magnitudes stimulated the Greeks to develop a more sophisticated theory of proportion. This culminated with the ideas of Eudoxus that ended up in Euclid. (The theory of Eudoxus has something in common with the modern theory of Dedekind cuts.)

3. In modern terms, two magnitudes are commensurable if and only if when we divide one by another we get what kind of number? (Note: Greek mathematicians at this time did not use fractions, but used ratios instead). So showing that there exists incommensurable lengths is similar to showing the existence of what sort of numbers?

4. Show that if \( d \) is the diagonal of a square, and if \( s \) is the side, then the ratio \( d : s \) is incommensurable. Note: this is often thought to be the first incommensurable ratio discovered, but another point of view is that the first incommensurable ratio involved the pentagon.

5. Is the above problem equivalent to showing that \( \sqrt{2} \) is irrational?

The Golden Section

1. Instead of \( \sqrt{2} \), the earliest known irrational number might have been the golden section. This number is related to \( \sqrt{n} \) for which \( n \)? The diagonal on a cube is related to \( \sqrt{n} \) for which \( n \)?

2. Note: I will cover the details of this in class when we talk about constructing the pentagon.

Paradoxes of Zeno

1. Zeno and the Eleatic philosophers believed which of the following? (i) all things change, and can be divided, or (ii) everything is indivisible and permanent and that change is just an illusion.

2. Zeno was trying to refute his opponents’ philosophy by showing that paradoxes emerge if we make the assumptions of his opponents. This is similar to which proof technique. [Hint: proof by c]

3. How many paradoxes did Zeno propound? (according to Aristotle)

4. Describe the paradox of Achilles (and the Tortoise). This is Zeno’s most famous paradox. (You can skim the others if you want).

5. According to Boyer (the author) what was the most far-reaching conclusion of this “Heroic Age”? (Hint: see the end of the section).
Deductive Reasoning

I have no study questions for this, so you can skim it. Personally, I like the idea that Thales was able to use deductive reasoning to prove theorems, but not as rigorously as later mathematicians.

Geometric Algebra

I have no study questions for this, so you can skim it. Some of geometric ideas described in this section pre-date the Greeks, so I find this section a bit misleading. For example the geometric view of the identity \((a + b)^2 = a^2 + 2ab + b^2\) is surely older than the Greeks. Also the notion of “Geometric Algebra” is controversial in history of mathematics circles. However, I will cover some of the ideas of this section when we talk about ruler and compass constructions.

Democritus of Abdera

1. What is Democritus best known for today? [hint: think chemistry] What was he also famous for in ancient times? [hint: g]
2. Where was he said to have travelled?
3. His critics claimed that he did not invent his atomism, but did what instead? [hint: L]
4. What two volume formulas were proved by Democritus? [Hint: py and co]
5. Did Archimedes think that Democritus’ proof was rigorous? Unfortunately we have no way of judging this since Democritus’ writings are lost.
6. What are the six problems that figured highly in the “Heroic Age”?