The strong chromatic index of subcubic planar graphs

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A strong edge-coloring of a graph $G$ is a proper edge-coloring with the additional restriction that each color class form an induced matching in $G$. The strong chromatic index of $G$ is the minimum $k$ for which $G$ has a strong edge-coloring using $k$ colors. Erdős and Nešetřil conjectured that every graph with maximum degree $\Delta$ has strong chromatic index at most $\frac{5}{4}\Delta^2$ if $\Delta$ is even, and at most $\frac{5}{4}\Delta^2 - \frac{1}{2}\Delta + \frac{1}{4}$ if $\Delta$ is odd. If true, both cases are best possible.

In 1990, Faudree, Gyárfás, Schelp, and Tuza revised this conjecture of Erdős and Nešetřil for planar graphs with maximum degree at most 3, stating that such graphs should have strong chromatic index at most 9. We verify this conjecture, which is best possible, and extend it to loopless multigraphs.

Much of this talk will be dedicated to the history of these conjectures and will end with a gentle sketch of our proof. Very little graph theory knowledge will be assumed in this talk, and many unresolved problems will be presented!

This is joint work with A.V. Kostochka, X. Li, W. Ruksasakchai, T. Wang, and G. Yu.