

**A CORRECTION TO “ULTRADIFFERENTIABLE  
FUNCTIONS ON LINES IN  $\mathbb{R}^n$ .” [PROC. AMER.  
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ABSTRACT. The proof of Theorem 1 in the original article is revised.

The proof of Theorem 1 in the article uses Lemma 3(ii) which turns out to be valid only for the case  $n = 2$ . In this note the proof of Theorem 1 is revised by replacing Lemma 3(ii) by the proposition stated below.

For  $x \in \mathbb{R}^n$ ,  $0 \neq \xi \in \mathbb{S}^{n-1}$ , let  $H_{x\xi}$  denote the hyperplane in  $\mathbb{R}^n$  that passes through  $x$  with  $\xi$  as its normal vector. If  $\varphi : \mathbb{R}^{n-1} \rightarrow \mathbb{R}^n$  is a linear map such that  $H_{x\xi} = \varphi(\mathbb{R}^{n-1})$ , then the restriction of a function  $u \in C^\infty(\mathbb{R}^n)$  to  $H_{x\xi}$  is defined by  $u_{x\xi}(\mathbf{t}) = u(x + \varphi(\mathbf{t}))$ ,  $\mathbf{t} \in \mathbb{R}^{n-1}$ .

**Proposition.** For any  $u \in C^\infty(\mathbb{R}^n)$ , the following inequality holds.

$$(0.1) \quad \max_{|\alpha|=k} |\partial^\alpha u(x)| \leq \max_{\beta \in \mathbb{Z}_+^{n-1}, |\beta|=k} \max_{|\xi|=1} |\partial^\beta (u_{x\xi})(0)|, \forall k \geq 0.$$

*Proof.* Conforming to the notation in the article, let  $a_1, \varepsilon \in \mathbb{R}$ ,  $\varepsilon > 0$ ,  $x \in \mathbb{R}^n$ ,  $k \in \mathbb{Z}$ ,  $k \geq 0$ , and  $(\alpha_1, \alpha_2, \dots, \alpha_n) \in \mathbb{Z}_+^n$ ,  $|\alpha| = k$ , be fixed. Put  $a_{1j} = a_1 + \frac{\varepsilon j}{k}$ ,  $j = 0, \dots, k$ . Consider the hyperplanes given by the images of the linear maps

$$\begin{aligned} \varphi_j & : \mathbb{R}^{n-1} \rightarrow \mathbb{R}^n, \\ \varphi_j(t_2, \dots, t_n) & = (x_1 + a_{1j}t_2, x_2 + t_2, \dots, x_n + t_n), \forall j, 0 \leq j \leq k. \end{aligned}$$

Then,

$$\partial_2^{\alpha_1 + \alpha_2} \partial^{\alpha''} (u \circ \varphi_j)(0) = \sum_{l=0}^{\alpha_1 + \alpha_2} a_{1j}^{\alpha_1} \left( \partial_1^l \partial_2^{\alpha_1 + \alpha_2 - l} \partial^{\alpha''} u \right) (x),$$

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where  $\partial^{\alpha''} = \partial_3^{\alpha_3} \dots \partial_n^{\alpha_n}$ . By applying Lemma 3(i), in the article, we have for  $\varepsilon = 8e^3$  and  $l = \alpha_1$ ,

$$|\partial^{\alpha} u(x)| \leq \max_{|\beta|=|\alpha|} \max_{0 \leq r \leq k} |\partial^{\beta} (u \circ \varphi_r)(0)|.$$

■

Let  $V$  be an  $n$ -dimensional vector space. By using a linear isomorphism between  $V$  and  $\mathbb{R}^n$ , the classes  $C^{\infty}(V)$  and  $C^M(V)$  can be identified with  $C^{\infty}(\mathbb{R}^n)$  and  $C^M(\mathbb{R}^n)$ , respectively. When  $n = 2$ , the inequality 0.1 reduces to the last inequality in the proof of Theorem 1. In particular, the conclusion of Theorem 1 is valid for any 2-dimensional vector space  $V$ . Hence, by the above proposition and by induction on  $n$ , we see that Theorem 1 holds for all  $n$ .

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