Syllabus: Math 410
Modern Geometry
Spring 2007: CRN 21182

Instructor: Dr. Wayne Aitken. Office: SCI2 327. Phone: 750-4155
E-mail: waitken@csusm.edu
Web: http://public.csusm.edu/aitken_html/

Meeting Time and Place: SCI2 306. MW 1:00 – 2:15 PM

Office Hours:

Monday	2:30 – 3:30 PM
Wednesday	12:00 – 12:50, 4:00 – 5:00 PM

Prerequisites: Experience with logic, mathematical proofs, and basic set theory. This experience can be obtained by taking Math 350 or Math 370 (with a grade of C or higher).

Required texts:
Euclidean and Non-Euclidean Geometry: Development and History
by Marvin Jay Greenberg
ISBN: 0716724464
Publisher: W. H. Freeman (3rd edition 1993)

Euclid's Elements
Translated from the Greek by Thomas L. Heath.

Optional texts:

Geometry: Euclid and Beyond
by Robin Hartshorne
ISBN: 0387986502
Publisher: Springer (2000)

Course Description: This is a course on Euclidean and non-Euclidean geometries with emphasis on (i) the contrast between the traditional and modern approaches to geometry, and (ii) the history and role of the parallel postulate. This course will be useful to students who want to teach and use Euclidean geometry, to students who want to learn more about the history of geometry, and to students who want an introduction to non-Euclidean geometry.

Euclid wrote the first preserved Geometry book, which has traditionally been held up as a role model for logical reasoning inside and outside mathematics for thousands of years. However, Euclid has several subtle logical lapses and flaws, and in the late 1800s it was necessary to revise the foundations of Euclidean geometry. In this course we will review the traditional approach, and then a modern approach based on Hilbert's axioms developed around 1900. The famous mathematician David Hilbert, building on work of several other mathematicians, was able to develop axioms that allow one to develop geometry without any overt or covert appeals to intuition. His idea was that, although intuitions are important in discovering, motivating, communicating and appreciating the theorems, rigorous proofs should not appeal to them. With the more modern approach to the axiomatic method that is not logically dependent on intuition, mathematicians are free to develop more types of geometries than the traditional geometry. We will discuss different types and models of geometry that are used today. These include finite geometries with applications in discrete mathematics and number theory, spaces of more than three dimensions, geometries whose coordinates are not real numbers, and geometries where a line can pass through a circle without actually intersecting the circle. Many of these geometries are useful, and not just curious examples.

A second major theme of the course will be the history and role of the parallel postulate. The parallel postulate makes the assumption that anytime you have a point P and a line l not going through P, there is one and only one line m going through P that is parallel to l (this is closely related to Euclid's original fifth postulate). Modern geometry began in the 1800's with the realization that there are interesting consistent geometries for which the parallel postulate is false. For example, hyperbolic and elliptic geometry do not satisfy the parallel postulate.

Since this postulate is less intuitively obvious than the other axioms of geometry, many mathematicians, especially medieval Arab mathematicians and later several European mathematicians of the 1700's, tried to make the parallel postulate a theorem and not an axiom. This goes along with the traditional idea that axioms should be restricted to a few simple, self-evident propositions, and the rest of the subject should be built upon these using proof. However, no mathematician was able to show that the parallel postulate followed as a theorem from the other axioms. Several prominent mathematicians thought that they had a proof of the parallel postulate, but subtle flaws were later discovered in their proofs. Finally mathematicians such as Lobachevsky and Bolyai started to believe that it is possible for there to be geometries where the parallel postulate fails, and they proved theorems about such *non-Euclidean* geometries. After the discovery of (Euclidean) models of non-Euclidean geometries in the late 1800's, no one was able to doubt the existence and consistency of non-Euclidean geometry. Also, these models show that the parallel postulate is independent of the other axioms of geometry: you cannot prove the parallel postulate from the other axioms.

Schedule:

Unit 1A. Classical Euclidean Geometry. We will spend two or three weeks proving theorems from the traditional viewpoint. Many of the theorems proved, and some of the proofs themselves, will be familiar from high-school geometry. We will study a translation of Euclid's original geometry, and will attempt to find any hidden assumptions that are made, or appeals to intuition instead of logic.

Unit 1B. Modern Approach to Axiomatics. We will study a version of Hilbert's axioms of incidence and betweenness and prove many of the theorems that were taken for granted by Euclid and others. We will show how these notions can be developed without appealing to our geometric intuitions. We will develop the idea of non-traditional models and types of geometry. The first test will cover Unit 1A and Unit 1B.

Unit 2. Modern Euclidean and Neutral Geometry. We will study the rest of Hilbert's axioms, and develop (some of) Euclidean geometry from the modern point of view. We will also discuss the role of the parallel postulate in Euclidean geometry. In particular we will investigate the question of whether or not the parallel postulate is necessary for geometry. We will discuss the history of this question, and will develop a geometry, called *neutral geometry*, to help us conduct a mathematical investigation of the question. Neutral geometry is essentially Hilbert's axioms without the parallel postulate. If you add the parallel postulate to neutral geometry you obtain Euclidean geometry. If you add the negation of the parallel postulate you obtain hyperbolic geometry. So all our proofs in neutral geometry will be valid in both Euclidean and non-Euclidean geometry. In neutral geometry it will be clear that there are several important statements in geometry that are equivalent to the parallel postulate, and we will see why it was originally difficult to imagine a geometry without the parallel postulate. For example, the statement that the angle sum of a triangle add up to 180 degrees will be shown to be equivalent to the parallel postulate. The second test will cover Unit 2.

Unit 3. Hyperbolic Geometry and Non-Euclidean Geometry. We will finish up neutral geometry. We will study the history of hyperbolic geometry. We will discuss some of its important theorems. Many of these follow from our work in neutral geometry. Some of the more advanced theorems, however, will be given with sketchy proofs. Models of hyperbolic geometry will be discussed, and we will justify the (relative) consistency of hyperbolic geometry. We will briefly discuss other non-Euclidean geometries. Finally we will briefly of how non-Euclidean geometry led to revolutionary ideas such as Einstein's theory of relativity, or new fields such as differential geometry. The final exam is comprehensive with an emphasis on Unit 3.

Learning Objectives:

1. To understand and be able to describe the difference between Euclid's approach to geometry and Hilbert's modern approach. To be able to explain the motivations of the modern approach and how the modern approach allows mathematicians to develop new geometries.

2. To become better at writing mathematical proofs.

3. To understand and be able to describe how the ideas of incidence and betweenness can be developed without appeal to intuitions, and in general how geometric proof can be freed of logical dependency on intuition.

4. To understand and be able to describe the history of the parallel postulate, and its role in the history of geometry.

5. To understand some non-Euclidean geometries, to be able to describe some models of non-Euclidean geometries, and to know several of the important theorems of hyperbolic geometry.

Grading: Your total grade will be based on three unit grades.

30%	Unit 1
30%	Unit 2
40%	Unit 3 and Comprehensive Final

Each unit grade will be based largely on your performance on the unit test: depending on the unit, 75 to 90 percent each unit grade will be determined by your score on the unit test; the rest of the unit grade will depend on homework, quizzes and participation.

Minimal Standards: Any student who scores less than 50% on any exam, or on any overall unit grade, cannot pass the course. Such students will not be allowed to take further exams.

Exams: Exam dates will be announced and posted on the course website at least two weeks in advance of the actual date. The final will be partially comprehensive. A preliminary, subject-to-change, schedule is as follows:

Unit 1 exam	Early March
Unit 2 exam	Mid April
Final incl. Unit 3	May 14 (11:30 - 13:30)

Unit Grades, Homework, Quizzes: Most of your unit grade (75%-90% depending on the unit) will be based on your performance on the exam for that unit. The rest, still a significant proportion, will be based on how you do on homework assignments, quizzes, and on the quality of your class participation.

Writing: Part of the grade for the homework assignments will be based on the quality and clarity of the writing displayed. Remember to write your work in complete, clear, and grammatically correct sentences. Of course, the work should be mathematically correct and clear as well.

Classroom Participation: You should behave in a manner that fosters your learning and that of your fellow students, and that creates a positive classroom environment. For example, you should volunteer to present assignments, and present it in a helpful manner. Do not use the computers for outside activities during class. Also avoid coming later or leaving early, and in general avoid behavior that distracts fellow students.

Make-Up Work: I will give make-up exams to students with excused absences. Make-up exams will often have an oral as well as a written component. Your lowest quiz or homework score will be dropped in each unit, so there will be no make-up quizzes. (Students with extended absences will be given make-up work to replace the quizzes, but only in the case of excused absences.)

Academic Honesty: You will be expected to adhere to standards of academic honesty and integrity, as outlined in the Student Academic Honesty Policy. For homework assignments, you are encouraged to discuss your ideas with classmates, but make sure that the work you turn in is your own. All ideas/material that are borrowed from other sources must have appropriate references to the original sources. Students are not allowed to help each other during examinations, nor are they allowed to use any non-approved aides or devices (including cell-phones, calculators, or iPods). If you believe there has been a violation of these guidelines by someone in the class, please bring it to my attention. I reserve the right to discipline any student for academic dishonesty, in accordance with the general rules and regulations of the university. Disciplinary action may include the lowering of grades or the assignment of a failing grade for an exam, assignment, or the class as a whole. Incidents of academic dishonesty will also be reported to the Dean of Students. Sanctions at the University level may include suspension or expulsion from the University.

ADA Policy: Students with disabilities who require reasonable accommodations must be approved for services by providing appropriate and recent documentation to the Office of Disabled Student Services (DSS). This office is located in Craven Hall 5205, and can be contacted by phone at (760) 750-4905, or TTY (760) 750-4909. Students authorized by DSS to receive reasonable accommodations should meet me during office hours in order to ensure confidentiality.