Syllabus: Math 410
Modern Geometry
Spring 2008: CRN 21906

Instructor: Dr. Wayne Aitken. Office: SCI2 327. Phone: 750-4155
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Meeting Time and Place: SCI2 306. MW 2:30 – 3:45 PM

Websites: We will use two websites. The first is the WebCT website. It contains study guides, test dates, announcements, grades, and other materials related to the management of the course. Its URL is

https://webct6.csusm.edu/webct/logon/162595763001

The second is a public website set up by the instructor. It contains handouts and other information related to the the course content. Its URL is

http://public.csusm.edu/aitken_html/m410/

Office Hours: These will be in SCI2 327

Monday	4:00 - 4:55 PM
Tuesday	11:00 - 11:55 AM
Wednesday	1:00 - 1:55 PM

These hours apply between January 22 and May 7th.

Prerequisites: Experience with logic, mathematical proofs, and basic set theory. This experience can be obtained by taking Math 350 or Math 370 (with a grade of C or higher).

Required texts:

Euclidean and Non-Euclidean Geometries: Development and History
by Marvin Jay Greenberg
ISBN: 0716799480
Publisher: W. H. Freeman and Company (4th edition 2008)

Euclid's Elements
Translated from the Greek by Thomas L. Heath.
This translation is no longer under copyright, so it can be obtained from several sources.

Optional texts:

There are several optional texts mentioned on the course website.

Course Description: This is a course on Euclidean and non-Euclidean geometries with emphasis on (i) the contrast between the traditional and modern approaches to geometry, and (ii) the history and role of the parallel postulate. This course will be useful to students who want to teach and use Euclidean geometry, to students who want to learn more about the history of geometry, and to students who want an introduction to non-Euclidean geometry.

Euclid wrote the first preserved Geometry book which has traditionally been held up as a role model for logical reasoning inside and outside mathematics for thousands of years. However, Euclid has several subtle logical omissions, and in the late 1800s it was necessary to revise the foundations of Euclidean geometry. The need for such a revision was partly due to advances in mathematical logic and changes in the conception of an axiom system. In this course we will review the traditional approach, and then a modern approach based on Hilbert's axioms developed around 1900. The famous mathematician David Hilbert, building on work of several other mathematicians, was able to develop axioms that allow one to develop geometry without any overt or covert appeals to intuition. His idea was that, although intuitions are important in discovering, motivating, communicating and appreciating the theorems, rigorous proofs should not appeal to them. With the more modern approach to the axiomatic method that is not logically dependent on intuition, mathematicians are free to develop more types of geometries than the traditional Euclidean geometry. We will discuss different types and models of geometry that are used today. These include finite geometries with applications in discrete mathematics and number theory, spaces of more than three dimensions, geometries whose coordinates are not real numbers, and geometries where a line can pass through a circle without actually intersecting the circle. Many of these geometries are useful, and not just curious examples.

A second major theme of the course will be the history and role of the parallel postulate. The parallel postulate makes the assumption that anytime you have a point P and a line l not going through P, there is one and only one line m going through P that is parallel to l (this is closely related to Euclid's original fifth postulate). Modern geometry began in the 1800's with the realization that there are interesting consistent geometries for which the parallel postulate is false. For example, hyperbolic and elliptic geometry do not satisfy the parallel postulate.

Since this postulate is less intuitively obvious than the other axioms of geometry, many mathematicians, especially medieval Arab mathematicians and later several European mathematicians of the 1700's, tried to make the parallel postulate a theorem and not an axiom. This goes along with the traditional idea that axioms should be restricted to a few simple, self-evident propositions, and the rest of the subject should be built upon these using proof. However, no mathematician was able to show that the parallel postulate followed as a theorem from the other axioms. Several prominent mathematicians thought that they had a proof of the parallel postulate, but subtle flaws were later discovered in their proofs. Finally mathematicians such as Lobachevsky and Bolyai started to believe that it is possible for there to be geometries where the parallel postulate fails, and they proved theorems about such *non-Euclidean* geometries. After the discovery of (Euclidean) models of non-Euclidean geometries in the late 1800's, no one was able to doubt the existence and consistency of non-Euclidean geometry. Also, these models show that the parallel postulate is independent of the other axioms of geometry: you cannot prove the parallel postulate from the other axioms.

Schedule:

Unit 1A. Classical Euclidean Geometry. We will spend about three weeks proving theorems from Euclid's Elements (Books I and III). Many of the theorems will be familiar from high-school geometry. We will critique Euclid's method, and discuss possible improvements.

Unit 1B. Modern Approach to Axiomatics. We will study a version of Hilbert's axioms of incidence and betweenness and prove many of the theorems that were taken for granted by Euclid and others. We will show how these notions can be developed without appealing to our geometric intuitions. We will develop the idea of non-traditional models and types of geometry. The first test will cover Unit 1A and Unit 1B.

Unit 2. Modern Euclidean and Neutral Geometry. We will study the rest of Hilbert's axioms, and develop (some of) Euclidean geometry from the modern point of view. We will also discuss the role of the parallel postulate in Euclidean geometry. In particular we will investigate the question of whether or not the parallel postulate is necessary for geometry. We will discuss the history of this question, and will develop a geometry, called *neutral geometry*, to help us conduct a mathematical investigation of the question. Neutral geometry is essentially Hilbert's axioms without the parallel postulate. If you add the parallel postulate to neutral geometry you obtain modern Euclidean geometry. If you add the negation of the parallel postulate you obtain hyperbolic geometry. So all our proofs in neutral geometry will be valid in both Euclidean and non-Euclidean geometry. The second test will cover Unit 2.

Unit 3. Hyperbolic Geometry and Non-Euclidean Geometry. We will finish up neutral geometry. We will then study the history of hyperbolic geometry, and some of its important theorems. Many of these follow from our work in neutral geometry. Some of the more advanced theorems, however, will be given with sketchy proofs. Models of hyperbolic geometry will be discussed, and we will justify the (relative) consistency of hyperbolic geometry. We will briefly discuss other non-Euclidean geometries. Finally we will briefly discuss how non-Euclidean geometry led to revolutionary ideas such as Einstein's theory of relativity as well as new fields of mathematics such as differential geometry. The final exam is comprehensive with an emphasis on Unit 3.

Learning Objectives:

1. To understand and be able to describe the difference between Euclid's approach to geometry and Hilbert's modern approach. To be able to explain the motivations of the modern approach and how the modern approach allows mathematicians to develop new geometries.

2. To become better at reading and writing mathematical proofs.

3. To understand and be able to describe how the ideas of incidence and betweenness can be developed without appeal to intuitions, and in general how geometric proof can be freed of a logical dependency on intuition.

4. To understand and be able to describe the history of the parallel postulate, and its role in the history of geometry.

5. To understand some non-Euclidean geometries, to be able to describe some models of non-Euclidean geometries, and to know several of the important theorems of hyperbolic geometry.

Grading: Your total grade will be based on three unit grades.

30%	Unit 1
30%	Unit 2
40%	Unit 3 and Comprehensive Final

Each unit grade will be based largely on your performance on the unit test: depending on the unit, 75 to 90 percent of your unit grade will be determined by your score on the unit test; the rest of the unit grade will depend on homework, quizzes and participation.

Minimal Standards: To pass the course, you are required to score at least 50% on every test, and in every unit grade. Students failing this requirement will not be permitted to take further exams. This is a necessary, but not sufficient condition for passing the course.

Exams: Exam dates will be announced and posted on the WebCT math 410 website at least two weeks in advance of the actual date. The final will be partially comprehensive. A preliminary schedule is as follows (the dates of the first two exams are subject to change):

Unit 1 exam	February 27
Unit 2 exam	April 16
Final incl. Unit 3	May 12 (4 - 6 PM)

Unit Grades, Homework, Quizzes: Most of your unit grade (75%-90% depending on the unit) will be based on your performance on the exam for that unit. The rest, still a significant proportion, will be based on how you do on homework assignments, quizzes, and on the quality of your class participation.

Writing: Part of the grade for the homework assignments, and some of your quizzes, will be based on the quality and clarity of the writing displayed. Remember to write your work in complete, clear, and grammatically correct sentences. Of course, the work should be mathematically correct and clear as well.

Classroom Participation: You should behave in a manner that fosters your learning and that of your fellow students, and that creates a positive classroom environment. For example, you should volunteer to present assignments, and present them in a helpful manner. Do not use the computers for outside activities during class. Also avoid coming late or leaving early, and in general avoid behavior that distracts fellow students.

Make-Up Work: I will give make-up exams to students with excused absences. Make-up exams may have an oral as well as a written component. Your lowest quiz or homework score will be dropped in each unit, so there will be no make-up quizzes or assignments. (Students with extended absences will, at the instructor's discretion, either be given make-up work to replace the quizzes, or have their exam grades count for a higher proportion of the affected unit grade.)

Academic Honesty: You will be expected to adhere to standards of academic honesty and integrity, as outlined in the Student Academic Honesty Policy. For homework assignments, you are encouraged to discuss your ideas with classmates, but make sure that the work you turn in is your own. All ideas/material that are borrowed from other sources must have appropriate references to the original sources. Students are not allowed to help each other during examinations, nor are they allowed to use any non-approved aides or devices (including cell-phones, calculators, or iPods). If you believe there has been a violation of these guidelines by someone in the class, please bring it to my attention. I reserve the right to discipline any student for academic dishonesty in accordance with the general rules and regulations of the university. Disciplinary action may include the lowering of grades or the assignment of a failing grade for an exam, assignment, or the class as a whole. Incidents of academic dishonesty will also be reported to the Dean of Students. Sanctions at the University level may include suspension or expulsion from the University.

ADA Policy: Students with disabilities who require reasonable accommodations must be approved for services by providing appropriate and recent documentation to the Office of Disabled Student Services (DSS). This office is located in Craven Hall 5205, and can be contacted by phone at (760) 750-4905, or TTY (760) 750-4909. Students authorized by DSS to receive reasonable accommodations should meet me during office hours in order to ensure confidentiality.

Key Dates:

Nov 30 - Jan 18: Schedule adjustment period.
Jan 19 (Sat): Official first day of the Spring 2008 semester.
Jan 21 (Mon): Martin Luther King, Jr. Day - Campus closed.
Jan 23 (Wed): First day of Math 410.
Jan 19 - Feb 1 (Fri): Add/drop period. Open University registration.
Feb 1 (Fri): Last day to apply for Fall 2008 UG graduation.
Mar 31 - Apr 5: Spring break (Mon to Sat).
May 7 (Wed): Last day of Math 410.
May 9 (Friday): Last day of classes for Spring 08.
May 10 (Sat) - 16 (Fri): Final exam period.
May 12 (Mon): Final exam for Math 410. 4-6 PM.
May 16 (Fri): Official last day of the Spring 2008 semester.
May 22 (Thurs): Grades due from instructors (by 3:00 pm).
Jun 1 (Sun): Grades available on SMART System (but check WebCT before then)